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VIBRATION ANALYSIS OF PIPING SYSTEMS

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VIBRATION ANALYSIS
OF PIPING SYSTEMS

by

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Submitted in partial fulfillment
for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

from the

UNITED STATES NAVAL POSTGRADUATE SCHOOL
May 1966

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ABSTRACT

A digital computer program, originally developed by George E. Fink, capable of determining the in-and/or out-of-plane vibration frequencies of a single plane piping system, using the basic transfer method, is modified and augmented by the writer. The program is modified to use the writer's method to reduce the amount of calculation and is augmented to use Marguerre and Uhrig's modifying frequency determinant method and Pestel and Mahrenholtz's remainder method for higher natural frequencies.

An analysis is made of difficulties encountered with the original transfer method and utilizing the modified methods.

The program accepts branched systems and non-stiff intermediate supports. A discussion and an explanation of how to use the program are included.

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CHAPTER I

INTRODUCTION

1-1 General remarks.

Today, in increasing measure, natural frequencies of a system capable of linear vibration are being determined with the help of the method of transfer matrices. This method is well explained in the recent book by E. Pestel and Leckie [3].*

This method which has been known for more than ten years, proceeds with particular simplicity especially with the use of digital computer. However, it meets with important numerical difficulties.

Numerical difficulties were observed by George E. Fink in his U. S. Naval Postgraduate School Master's thesis "Vibration analysis of piping systems" [4]. He used the straight-forward method of transfer matrices for calculation of the natural frequencies of vibrating piping systems.

The purpose of this paper is to deal with numerical difficulties and with the remedies which have been developed and also to investigate reduction in calculation.

1-2 Scope of work.

Numerical difficulties are encountered at higher natural frequencies and for systems having very stiff exterior springs. In examining these difficulties the paper by Marguerre and Uhrig was encountered [2]. This tends to explain difficulties both with the original transfer method and with the Pestel and Mahrenholtz's modified methods [1]. Although, Marguerre and Uhrig suggest several different methods for dealing with

*Numbers in brackets refer to the bibliography, page 37.

the difficulties, those which appear to be certain to eliminate the difficulties are so far outside the spirit of the transfer method that the advantage of this method would be lost. Therefore, it was decided to proceed with the transfer method to get as much good from it as one could.

The writer set out to investigate the remedies for failure at high frequency. The digital computer program "VIPIPE" presented in the Appendices was used to determine the natural frequencies of in-and out-of-plane vibration of a planar piping system based on the transfer method. This program was originally developed by Fink [4], but has been modified and augmented by the writer.

1-3 Assumptions and limitations.

1. System is conservative.
2. Distributed mass or/and lumped mass model is used. For the lumped mass model used in curved section (or in straight section) sufficient subsectioning is carried out to give acceptable answer.
3. The transfer method surely fails if the system has rigid exterior elements. The hangers are introduced as a support by a point matrix with appreciable linear and/or torsional compliance.
4. Program VIPIPE is limited to a system of a maximum of fifty components, twenty branches, and twenty hangers. It can only determine in-and/or out-of-plane natural frequencies of planar system.
5. Boundary conditions should be such that half of the state quantities are vanishing.*

*Refer to Appendix B.

1-4 Notation.

$[\quad]$	matrix.
$\{ \quad \}$	column matrix.
X, Y, Z	cartesian right handed coordinate system.
u, v, w	displacement in the XYZ direction respectively.
δ, ψ, ϑ	rotation about XYZ axis respectively.
M	bending moment.
N	normal force.
T	torque.
V	shear force.
Z_i	state vector at location i .
$[Z_i]$	state matrix at location i .
$[U_{i+i}]$	transfer matrix from i to $i + 1$.
$\Delta(\omega)$	frequency determinant.
$\Delta_m(\omega)$	modified determinant.*
$[N]$	non-vanishing state matrix of Z_0 .
u_{ij}	element of $[U]$.
$[S]$	spring matrix.
$[S]$	purged frequency determinant in the P-M method.*
$[I]$	unit matrix.
$[O]$	null matrix.
d	displacement vector.
$[d]$	displacement matrix.
p	internal force vector.
$[p]$	internal force matrix.

*In the text, the following abbreviations are used. M-D method stands for Modified Delta method and P-M method for Pestel and Mahrenholtz's remainder method. These will be explained in what follows.

$[G]$	coordinate transfer matrix.
$[R1], [R2], [R3]$	square sub-matrices.
V_0	column vector of every elements value of 1. (cf. p. 16).
$R(\omega)$	remainder.
X	correction factor vector.
P_0, P_1, P_2	assumed state quantities at the starting boundary.
X_0, X_1, X_2 Y_0, Y_1, X_1, X_2	correction factors in the P-M method.
D_1, D_2	purging factors in the M-D method.
λ	eigenvalue.
x	eigenvector.

CHAPTER II

TRANSFER METHODS AND NUMERICAL DIFFICULTIES

2-1 The state vector and transfer matrix.

A. State vector.

The state of vibration at location i , specified by state quantities, is described by the state vector Z_i . This vector is a column matrix whose elements completely describe the instantaneous displacements, both rectilinear and angular, of that point from its quiescent position, as well as the corresponding rectilinear and angular forces in the member at the same point and at the same instant, i.e., those forces, if a cutting plane were passed through the point at the instant of interest, would have to be applied to each cut face to prevent relative motion between them.

In the case of planar piping system, the state vector has six elements, three pertaining to displacement, and three pertaining to force.

The state vectors, for the in- and out-of-plane cases respectively, as used in VIPIPE, are:

$$Z_{ip} = \begin{cases} u & \text{displacement in X direction.} \\ v & \text{displacement in Y direction.} \\ \psi & \text{slope in X-Y plane with respect to X axis.} \\ M_z & \text{moment about Z axis.} \\ V_y & \text{shear in Y direction.} \\ N & \text{normal force in X direction.} \end{cases} \quad (2-1-1)$$

$$Z_{OP} = \begin{cases} \delta & \text{twist about X axis.} \\ T & \text{torque about X axis.} \\ -w & \text{(negative of) deflection in Z direction.} \\ \psi & \text{slope in X-Y plane about X axis.} \\ M_y & \text{moment about Y axis.} \\ V_z & \text{shear in Z direction.} \end{cases} \quad (2-1-2)$$

Thus, the state vector in this application is of order 6×1 , while the system state matrix* is 6×3 and the transfer matrix of a section is 6×6 ; however, in developing theory we will use the more general case $2r \times 1$, $2r \times r$, and $2r \times 2r$ respectively. A local XYZ right handed cartesian coordinate system and the elements of the state vector are arranged as in Fig. 2-1-1.

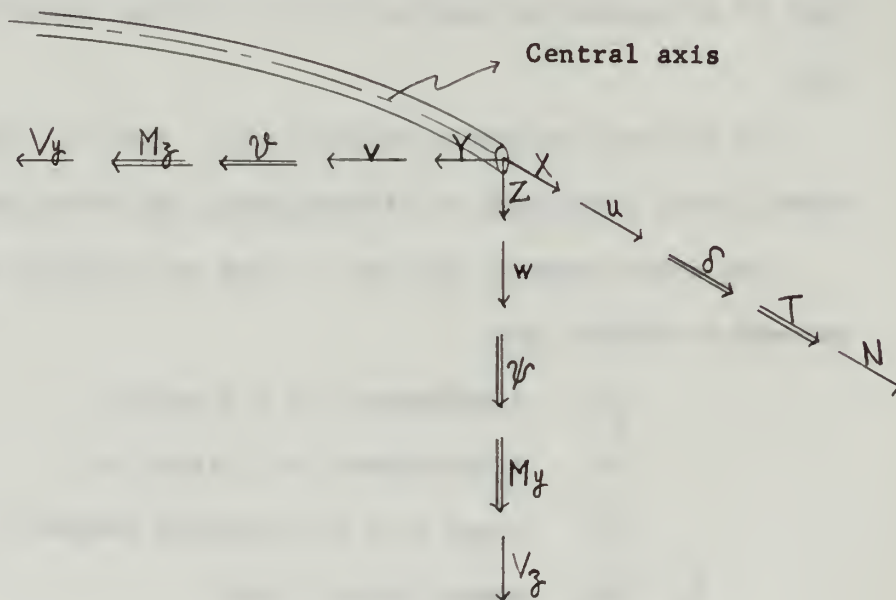


Fig. 2-1-1

B. Transfer matrix and transfer method.

From the state vector at location 1 (node 1), the state

*The concept of "state matrix" is introduced in Section 2-3.

vector at location $i + 1$ is given by

$$Z_{i+1} = [U_{i+1}] \cdot Z_i \quad (i = 0, 1, 2, \dots, n) \quad (2-1-4)$$

Here, matrix $[U_{i+1}]$ relating the state vectors of node i and node $i + 1$ is termed a transfer matrix.

We number the node $i + 1$ immediately following the node i from the left end, the beginning of the main system. From the recurrence relation 2-1-4, one easily obtains

$$\begin{aligned} Z_n &= [U_n] [U_{n-1}] \cdot \cdot \cdot [U_3] \cdot [U_2] \cdot [U_1] \cdot Z_0 \\ &= [U] \cdot Z_0 \end{aligned} \quad (2-1-5)$$

which is a linear relationship between the state quantities at location 0 and n of the system (boundaries of the main system). We call this method the transfer method.

Since of the $2r$ state quantities at each boundary r quantities are vanishing, only r are established by the boundary condition at location 0, and r homogeneous equations are established by the boundary condition at location n . The natural frequencies of the system are given by the zeros of the determinant of the corresponding r homogeneous equations. Since the application of interest is to the determination of the natural frequencies of piping systems, it would be instructive to illustrate the origin and nature of frequency dependence in transfer matrices. For this end, let us construct a point matrix for the in-plane case connecting Z_i^L with Z_i^R (Z_i^R and Z_i^L are state vectors of right side and left side, respectively, of a point mass m_i at node i).

To begin we assume that all of the elements of the state vectors vary with time, each having the factor $\cos \omega t$.

Noting that the deflection, slope, and moment* are continuous across the concentrated mass m_i , so that

$$u_i^R = u_i^L, \quad v_i^R = v_i^L, \quad \psi_i^R = \psi_i^L, \quad M_i^R = M_i^L$$

From the free body diagram of Fig. 2-1-2A, B, the following relations are established.

$$u(t) = u \cdot \cos \omega t \quad \frac{d^2 u(t)}{dt^2} = -u \omega^2 \cos \omega t \quad (2-1-6)$$

$$v(t) = v \cdot \cos \omega t \quad \frac{d^2 v(t)}{dt^2} = -v \omega^2 \cos \omega t \quad (2-1-7)$$

$$V_i^R \cdot \cos \omega t - V_i^L \cdot \cos \omega t = -m \cdot \frac{d^2 v(t)}{dt^2} \quad (2-1-8)$$

$$N_i^R \cdot \cos \omega t - N_i^L \cdot \cos \omega t = -m \cdot \frac{d^2 u(t)}{dt^2} \quad (2-1-9)$$

and substituting Eqn. 2-1-6 into Eqn. 2-1-8 and Eqn. 2-1-7 into Eqn.

2-1-9 we have

$$V_i^R = V_i^L + m v \omega^2 \quad (2-1-10)$$

$$N_i^R = N_i^L + m u \omega^2 \quad (2-1-11)$$

Now, the relation of the state vectors of the right and left side of the point mass m in matrix form is:

$$Z_i^R = [U_p] \cdot Z_i^L$$

i.e.,

$$\begin{bmatrix} u \\ v \\ \psi \\ M_z \\ V_y \\ N \end{bmatrix}_{iR} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & m \omega^2 & 0 & 0 & 1 & 0 \\ m \omega^2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} u \\ v \\ \psi \\ M_z \\ V_y \\ N \end{bmatrix}_{iL}$$

$[U_p]$ is a point matrix for in-plane vibration. It neglects any rotational inertia that the point mass may have; however, rotational inertia could

*The moment is continuous only if there are no point elements having finite rotational inertia.

be accounted for by including another nonzero term in the matrix.

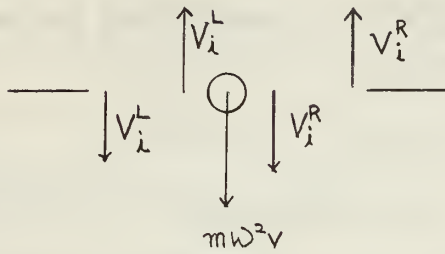


Fig. 2-1-2A

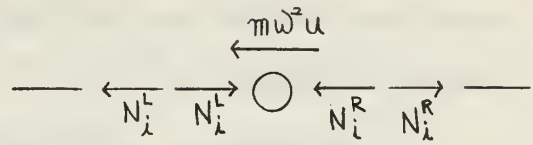


Fig. 2-1-2B

The derivation of transfer matrices is treated in great detail in reference [3] and transfer matrices for various components of planar piping system are incorporated in VIPIPE.

2-2 Failure of transfer method.

For higher natural frequencies and for systems having very stiff exterior springs numerical difficulties are encountered. In these cases, the numerical value of the frequency determinant is given as the difference between large numbers. So practically the natural frequencies can no longer be determined.

From a mathematical point of view, in case the transfer matrix $[U]$ has a single dominant real eigenvalue, the columns of $[U]^k$ become more and more parallel as k increases, and approach the first eigenvector x_1 [6],[2]. In this case the transfer matrix degenerates.

$$[U]^k x_1 = \lambda_1 x_1 \quad (2-2-1)$$

This could be the case for the lumped mass system as the number of lumped masses increases. This was observed in the straight section lumped mass system, but for the curved lumped mass system this was not observed.

As the frequency increases (elements of some transfer matrices are functions of frequency), the same phenomenon was observed in the distributed mass straight section transfer matrix. For example, for the sample problem of Appendix E-2-1,

At frequency of 20 rad/sec,

the largest eigenvalue: $\lambda_1 = .11484022 \times 10^2$

second largest eigenvalue: $\lambda_2, \lambda_3 = .99980414 \pm j0.01979160$

transfer matrix [U] is:

.99980413	0	0	0	0	.90535567 ^{-5*}
0	.25105587 ¹	.25986435 ³	.22549093 ⁻²	.14255229 ⁰	0
0	.30840073 ⁻¹	.25105587 ¹	.26651055 ⁻⁴	.22549093 ⁻²	0
0	.47565399 ⁴	.30070198 ⁶	.25105587 ¹	.25986435 ³	0
0	.56219618 ²	.47565399 ⁴	.30840073 ⁻¹	.25105589 ¹	0
-.43265609 ²	0	0	0	0	.99980413

*⁻⁵ means 10^{-5} , etc.

In this case, that is, at this frequency, we can see that there is no phenomenon of the columns becoming parallel. As the frequency increases this single dominant real eigenvalue keeps increasing, and at frequency of 1700 rad/sec.

the largest eigenvalue: $\lambda_1 = .5937220 \times 10^{10}$

second largest eigenvalue: $\lambda_2, \lambda_3 = -.11136847 \pm j.99377918$

transfer matrix [U] is:

-.11136847	0	0	0	0	.53482189 \backslash^{-5}
0	.14843074 \backslash^{10}	.13191200 \backslash^{11}	.12023306 \backslash^5	.10685242 \backslash^6	0
0	.16701804 \backslash^9	.14843074 \backslash^{10}	.13528929 \backslash^4	.12023306 \backslash^5	0
0	.18324149 \backslash^{15}	.16284870 \backslash^{16}	.14843074 \backslash^{10}	.13191200 \backslash^{11}	0
0	.20618799 \backslash^{14}	.18324149 \backslash^{15}	.16701804 \backslash^9	.14843074 \backslash^{10}	0
-.18465906 \backslash^6	0	0	0	0	.11136847

Normalized values of each column of [U] are:

.6031032 \backslash^{-7}	0	0	0	0	-.4802273 \backslash^{-5}
0	.8100280 \backslash^{-6}	.8100279 \backslash^{-6}	.8100280 \backslash^{-6}	.8100280 \backslash^{-6}	0
0	.9114641 \backslash^{-7}	.91146407 \backslash^{-7}	.9114641 \backslash^{-7}	.9114641 \backslash^{-7}	0
0	1	1	1	1	0
0	.1125225 \backslash^{-1}	.1125225 \backslash^{-1}	.1125225 \backslash^{-1}	.1125225 \backslash^{-1}	0
1	0	0	0	0	1

We can see several of the columns are almost parallel to each other. From this, it is easy to see the numerical difficulties we should encounter. This depends on the nature of the transfer matrix. For systems made of various sections, multiplications of different transfer matrices may result in the difficulties taking care of themselves.

2-3 Reduction in calculation in matrix multiplication.

In equation 2-1-5, $[U] = [U_n] \cdot [U_{n-1}] \dots [U_2] \cdot [U_1]$, so we should perform matrix multiplication of $2r \times 2r$ matrix times $2r \times 2r$ matrix successively.

Now using the fact of the boundary condition, introduce Z_0 as $2r \times r$ matrix called the "state matrix." This is formed from the state

vector as follows, all elements of the state matrix are zero except that in its i th column, unity appears in the same position as the i th generally nonzero term reading down the state vector. Then we can multiply $2r \times 2r$ matrices by $2r \times r$ matrices and thus can reduce the calculation considerably.

For example, in the in-plane case, for a built-in left end

$$Z_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The frequency determinant is obtained in the usual way from the boundary conditions at the right end.

We use the term "Delta method" to refer to this method which was discovered by the writer during the course of this study. This procedure is used in what follows instead of the usual procedure of the transfer method.

2-4 Pestel and Mahrenholtz's remainder method.

Instead of the frequency determinant, Pestel and Mahrenholtz introduced a remainder [1],[3]. This method, presenting the system state matrix as $2r \times r$ matrix, reduces the amount of calculation considerably compared to the original transfer method, and claims greater accuracy at higher frequencies because the remainder, which is the difference between small numbers, is itself small. This will be explained later.

A. Remainder.

For the non-vanishing state quantities of the state vector Z_0

which depends on frequency ω we use estimated values which approximate the true values, following which the Z_i ($i = 1, 2, 3, \dots, n$) are determined.

In the state vector Z_0 , since for eigenvibration only the ratios of state quantities are of interest, one of the non-vanishing state quantities is assigned the value of 1.

For example, for the in-plane case, at certain frequency, a cantilevered pipe which is free at the left end and fixed at the right end has:

$$Z_0^0 = \begin{bmatrix} 1 \\ P_1^0 + X_1^0 \\ P_2^0 + X_2^0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (2-4-1')$$

(in what follows, superscripts 0 and 1 denote the round of calculation; i.e., super-script 0 means first round of calculation with approximated state quantities in Z_0 , and superscript 1 means second round of calculation with improved state quantities in Z_0). In equation 2-4-1', we assigned the value of 1 to the non-vanishing state quantity in the first row of Z_0 , and the quantities of P_1^0 , P_2^0 are the previously chosen approximations and X_1^0 , X_2^0 are correction factors which are unknowns.

The state vector Z_0^0 of (2-4-1') in other matrix form is:

$$Z_0^0 = \begin{bmatrix} 1 & 0 & 0 \\ P_1^0 & 1 & 0 \\ P_2^0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ X_1^0 \\ X_2^0 \end{bmatrix} \quad (2-4-1'')$$

For the actual matrix multiplication Z_0° of (2-4-1'') is used. After a round of calculation, the approximated state quantities P_1° , P_2° will be improved in the form

$$\begin{aligned} P_1^I &= P_1^{\circ} + \chi_1^{\circ} \\ P_2^I &= P_2^{\circ} + \chi_2^{\circ} \end{aligned} \quad (2-4-2)$$

where χ_1° and χ_2° are corrections obtained by this process; the difference between χ_1° and X_1° , χ_2° and X_2° will be shown later.

Then, the improved state vector Z_0^I will be:

$$Z_0^I = \begin{bmatrix} 1 \\ P_1^I \\ P_2^I \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (2-4-3)$$

When we start first, usually P_1° , P_2° are set equal to 1, in the absence of better information. From the same method as (2-1-5) using Z_0° of (2-4-1'').

$$Z_n^{\circ} = [U_n] \cdot [U_{n-1}] \dots [U_2] \cdot [U_1] \cdot Z_0^{\circ}$$

From the boundary condition at location $n, 1, \dots$, from the three vanishing elements of Z_n , we get

$$0 = \Delta(\omega) \cdot [N^{\circ}] \cdot X^{\circ} = [S] \cdot X^{\circ} \quad (2-4-4)$$

$[N]$: nonvanishing sub-matrix of $[Z_0]$.

X : correction factor vector.

$\Delta(\omega)$: frequency determinant.

$[S]$: $\Delta(\omega) \cdot [N]$

For the cantilevered pipe of the example in the in-plane case, the frequency determinant $\Delta(\omega)$, see Ref. [3], consists of the top left

3x3 submatrix of the transfer matrix [U] .

Equation 2-4-4 may be written:

$$\begin{aligned}
 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} &= \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ P_1^0 & 1 & 0 \\ P_2^0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ X_1^0 \\ X_2^0 \end{bmatrix} \\
 &= \begin{bmatrix} u_{11} + u_{12}P_1^0 + u_{13}P_2^0 & u_{12} & u_{13} \\ u_{21} + u_{22}P_1^0 + u_{23}P_2^0 & u_{22} & u_{23} \\ u_{31} + u_{32}P_1^0 + u_{33}P_2^0 & u_{32} & u_{33} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ X_1^0 \\ X_2^0 \end{bmatrix} \\
 &= \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ X_1^0 \\ X_2^0 \end{bmatrix} \quad (2-4-5)
 \end{aligned}$$

In equation 2-4-5, the first column of [S] no longer contains large numbers, but the second and third columns remain unchanged. We call this "purging" the first column. From equation 2-4-5 we may write,

$$- \begin{bmatrix} s_{11} \\ s_{21} \\ s_{31} \end{bmatrix} = \begin{bmatrix} s_{12} & s_{13} \\ s_{22} & s_{23} \\ s_{32} & s_{33} \end{bmatrix} \cdot \begin{bmatrix} X_1^0 \\ X_2^0 \end{bmatrix} \quad (2-4-6)$$

Equation 2-4-6 has first, three equations with two unknowns (two correction factors), and second it has the purged column to the left and unpurged columns as coefficient matrix.

Equation 2-4-6 is the same as;

$$- \begin{bmatrix} u_{11} \\ u_{21} \\ u_{31} \end{bmatrix} = \begin{bmatrix} u_{12} & u_{13} \\ u_{22} & u_{23} \\ u_{32} & u_{33} \end{bmatrix} \cdot \begin{bmatrix} P_1^0 + X_1^0 \\ P_2^0 + X_2^0 \end{bmatrix} \quad (2-4-7)$$

From 2-4-7 we choose any two equations. If we choose the first two, then by Cramer's rule;

$$X_1^{\circ} = \frac{-\begin{vmatrix} u_{11} & u_{13} \\ u_{21} & u_{23} \end{vmatrix}}{\begin{vmatrix} u_{12} & u_{13} \\ u_{22} & u_{23} \end{vmatrix}} - P_1^{\circ} = \frac{\Delta_2}{\Delta_1} - P_1^{\circ} \quad X_2^{\circ} = \frac{-\begin{vmatrix} u_{12} & u_{11} \\ u_{22} & u_{21} \end{vmatrix}}{\begin{vmatrix} u_{12} & u_{13} \\ u_{22} & u_{23} \end{vmatrix}} - P_2^{\circ} = \frac{\Delta_3}{\Delta_1} - P_2^{\circ} \quad (2-4-8)$$

The Δ 's are defined by the preceding expressions. If $\omega = \omega_k$, where ω_k is a natural frequency of the system, then the third equation will be satisfied if we substitute these values for X_1° and X_2° . Otherwise, substituting the above value for X_1° , we get, from the third equation, a somewhat different value for X_2° ; we denote this by the symbol Y_2° .

$$Y_2^{\circ} = -\frac{u_{32} \Delta_2}{u_{33} \Delta_1} - \frac{u_{31}}{u_{33}} - P_2^{\circ} \quad (2-4-9)$$

We define the remainder $R^{\circ}(\omega)$ to be the difference between these values.

$$R^{\circ}(\omega) = X_2^{\circ} - Y_2^{\circ} = \frac{\Delta_3}{\Delta_1} + \frac{u_{32} \Delta_2}{u_{33} \Delta_1} + \frac{u_{31}}{u_{33}} \quad (2-4-10)$$

As we see from equation 2-4-10, for fixed ω , the remainder is independent of the assumed state quantities P_1° and P_2° . If the remainder is zero, $\omega = \omega_k$ since all the prescribed boundary conditions will be satisfied simultaneously.

For the next round of calculation for the same frequency, form the arithmetic average of the two values X and Y for the correction factor χ , i.e.,

$$\chi_1^{\circ} = X_1^{\circ} \quad \chi_2^{\circ} = \frac{1}{2}(X_2^{\circ} + Y_2^{\circ})$$

Then, new starting values are:

$$\begin{aligned}
P_1' &= P_1^0 - \chi_1^0 = \frac{\Delta_2}{\Delta_1} \\
P_2' &= P_2^0 - \chi_2^0 = \frac{1}{2} \left\{ \frac{\Delta_3}{\Delta_1} - \left(\frac{u_{32} \Delta_2}{u_{33} \Delta_1} + \frac{u_{31}}{u_{33}} \right) \right\} \quad (2-4-11)
\end{aligned}$$

which are also independent of P_1^0 and P_2^0 . A repeated round of calculation gives

$$\begin{aligned}
\chi_1' &= \chi_1' = -P_1' - \frac{\Delta_2}{\Delta_1} = 0 \\
\chi_2' &= \frac{1}{2} (\chi_2' - Y_2') = 0 \\
R^1(\omega) &= \chi_2' - Y_2' = R^0(\omega)
\end{aligned}$$

therefore,

$$\chi_2' = -Y_2' = \frac{1}{2} R^0(\omega)$$

Now, we see that the iteration is completed after the first round of calculation and the remainder is no longer the difference between very large numbers, because the remainder is of the same order of magnitude as the corrections χ_2' and Y_2' . During the iteration seeking the natural frequencies, we calculate $R^0(\omega_1)$ and $P_i^1(\omega_1)$'s ($i=1,2$), and increase frequency by an amount $\Delta\omega$ to get $\omega_2 = \omega_1 + \Delta\omega$. Then get $R^0(\omega_2)$ using $P_i^0(\omega_2) = P_i^1(\omega_1)$ ($i = 1,2$). The above example is for the case $r = 3$. In other cases for $r > 1$, the spirit is the same (in case $r = 1$, we cannot use the P-M method).

B. Various ways to get remainder.

There are many ways to calculate a remainder. Each one has a different value at the same frequency ($\omega \neq \omega_k$), but passes through zero at $\omega = \omega_k$.

Some fail and some give good results (see section 2-4-D).

The possible ways arise from the following choices;

1. Which one of the nonvanishing state quantities will be given the value of 1 in the state vector Z_0 .

2. Among the r homogeneous equations, as in equation 2-4-5, which $r-1$ equations will be chosen to get $X_1^0, X_2^0, \dots, X_{r-1}^0$.

3. Which of the X_i^0 's ($i = 1, 2, \dots, r-1$) in the r th equation is to be regarded as Y^0 and solved for in terms of the other X^0 's.

We will call these different ways by the name "sub-procedures."

C. The relation between a remainder and the frequency determinant.

From the equation 2-4-10, the remainder is

$$R(\omega) = \frac{u_{33}\Delta_3 + u_{32}\Delta_2 + u_{31}\Delta_1}{u_{33}\Delta_1}$$

From equation 2-4-8, we see that Δ 's are the minors of an element of the frequency determinant

$$\Delta_1 = M(u_{31}), \quad \Delta_2 = M(u_{32}), \quad \Delta_3 = M(u_{33})$$

where $M(u_{31})$ means the minor of u_{31} .

The remainder is

$$R(\omega) = \frac{\Delta(\omega)}{u_{33}\Delta_1}$$

where $\Delta(\omega)$ is seen to be the frequency determinant.

Expressing a remainder in general in terms of the frequency determinant;

$$R(\omega) = \frac{\Delta(\omega)}{u_{rt}\Delta_{rs}} \quad (2-4-12)$$

u_{rt} : coefficient element corresponding to Y^0 of r th row.

Δ_{rs} : minor of u_{rs} in $\Delta(\omega)$.

u_{rs} : element in r th row and the column corresponding to state quantity of 1 in Z_0 .

D. Failure of the P-M method.

In case the denominator of the remainder is zero, the P-M method certainly fails.* (Refer to equation 2-4-10 and 2-4-12). When the determinant in the denominator becomes nearly equal to zero or changes sign passing through zero, the remainder is difficult to evaluate: at certain frequencies the remainder may show an infinite or jumping phenomenon.

However, this does not necessarily show up at the same frequency with different sub-procedures, because this phenomenon depends solely on the nature of the transfer matrices.

Figure 2-4-1 and Figure 2-4-2 show that while one sub-procedure shows an infinite phenomenon at certain frequency, another sub-procedure does not show the same phenomenon at this frequency.

Pestel and Mahrenholtz illustrated their method with the case of a homogeneous beam ($2r = 4$). It leads to very good results for the bending frequencies. The reason is, because $r = 2$, there is no phenomenon of indefiniteness causing failure mentioned above and due to purging effect. The P-M method does show improvement also for the case of r greater than two.

For sufficiently high frequencies, the P-M method fails. The reason is as follows, if the columns have already become parallel, purging does not do any good, and if round-off error reaches critical point, where remainder is no longer reliable, this method fails.

*When this is the case, program VIPIPE skips to the next sub-procedure.

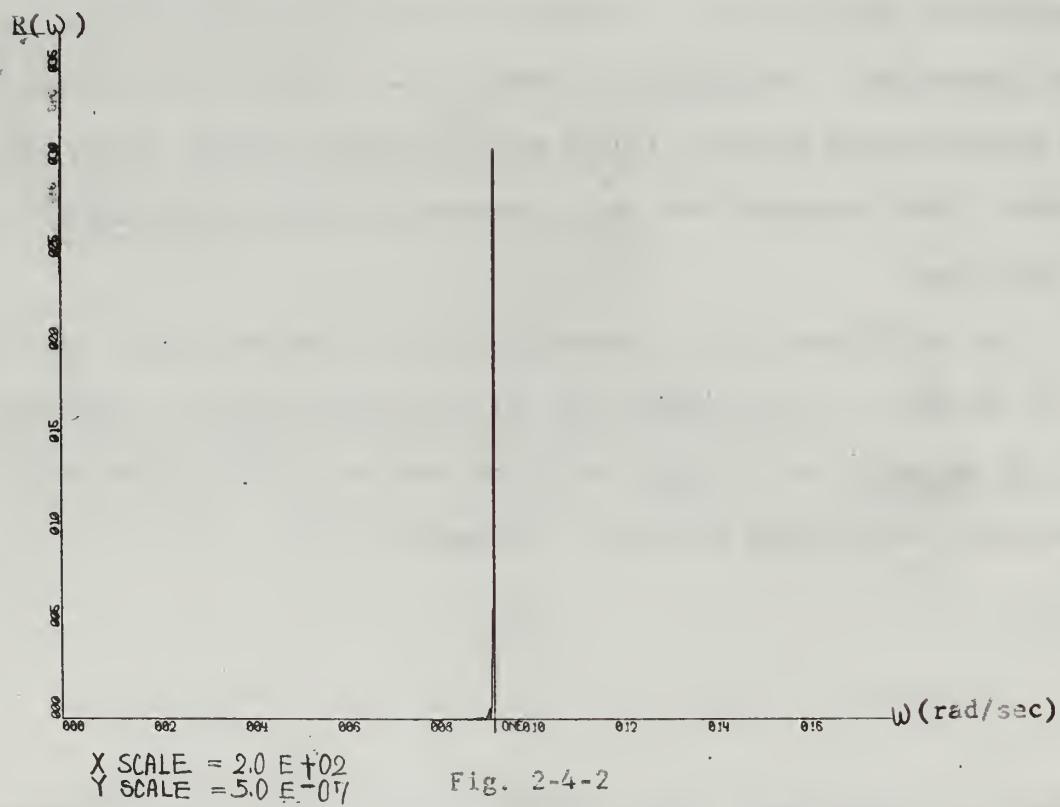
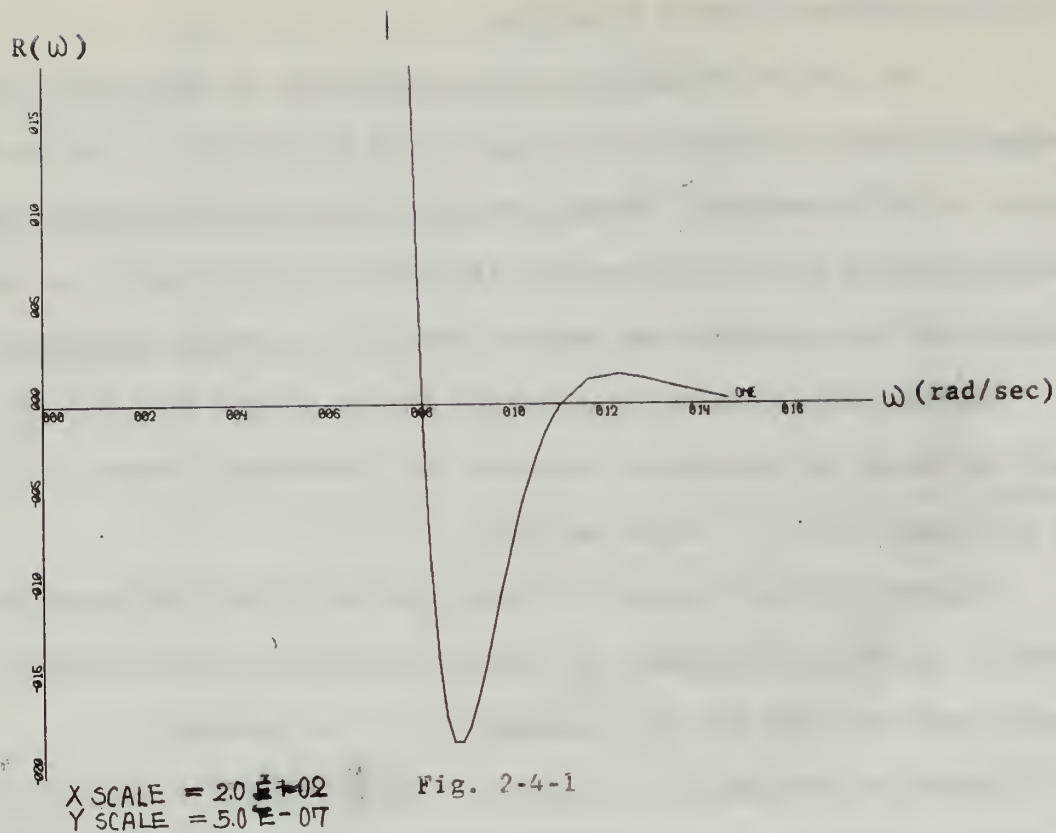


Figure 2-4-1 shows fourth and fifth mode frequencies of System 5* at frequency 826.88281950 rad/sec, and 1126.83799669 rad/sec (P-M method-subprocedure A with double precision arithmetic). Figure 2-4-2 shows infinite phenomenon at frequency 939.00298809 rad/sec while Figure 2-4-1 does not show this phenomenon at this Frequency. And Figure 2-4-2 has print out-put with mode frequency 826.88281915, also with 939.00298809 rad/sec (P-M method-subprocedure B with double precision arithmetic). From Figure 2-4-2, we see that the latter is a false mode frequency and the former does not show upon the graph due to the infinite phenomenon of the latter, but we know that there is a mode frequency at 826.88281915 rad/sec (this also can be checked with the frequency and remainder value print output).

2-5 Modified Delta method.

Instead of purging just one column vector among r columns (as in the P-M method) of frequency determinant, Marguerre and Uhrig showed in their paper [2] a possible way of extending the purging concept for problems greater than 4th order.

Through exactly the same method as Delta, we get a frequency determinant. At high frequency, because the columns approach parallelism, the value of the frequency determinant tends toward zero. If the columns are exactly parallel to each other, then of course all methods fail. If they become nearly parallel, then with a finite capacity of handling numbers, the results become meaningless.

Using Maguerre and Uhrig's purging concept, with the help of one column of the frequency determinant, obtain $r-1$ purging factors to purge

*See Appendix E-3.

another $r-1$ columns.* For the case of sixth order ($r=3$) as used in program VIPIPE, the frequency determinant at a certain frequency is:

$$\Delta(\omega) = \begin{vmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{vmatrix}$$

Purging factors got with the help of the third column of $\Delta(\omega)$ are

$$\begin{aligned} D_1 &= \frac{1}{3} \left(\frac{u_{11}}{u_{13}} + \frac{u_{21}}{u_{23}} + \frac{u_{31}}{u_{33}} \right) \\ D_2 &= \frac{1}{3} \left(\frac{u_{12}}{u_{13}} + \frac{u_{22}}{u_{23}} + \frac{u_{32}}{u_{33}} \right) \end{aligned} \quad (2-5-1)$$

Modifying the non-vanishing sub-matrix of the starting state matrix $[Z_0]$, we have

$$[N] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -D_1 & -D_2 & 1 \end{bmatrix} \quad (2-5-2)$$

Then, for the same frequency, repeat a round of calculation to get the modified frequency determinant.

$$\Delta_m(\omega) = \begin{vmatrix} u_{11} & D_1 u_{13} & u_{12} & D_2 u_{13} & u_{13} \\ u_{21} & D_1 u_{23} & u_{22} & D_2 u_{23} & u_{23} \\ u_{31} & D_1 u_{33} & u_{32} & D_2 u_{33} & u_{33} \end{vmatrix} \quad (2-5-3)$$

From the properties of determinants, $\Delta(\omega)$ is the same as $\Delta_m(\omega)$.

But the first and second columns of $\Delta_m(\omega)$ are not the same as those of $\Delta(\omega)$ any more. If the columns of the original frequency determinant were exactly parallel, the modified columns (first and second columns) of the modified frequency determinant will turn out to be all

*We use the term "Modified Delta method" to refer to this method.

zeros. In case the columns were nearly parallel, the first and second columns will turn out to be very small numbers, i.e., we will have purged two columns. So we can expect a more accurate value of the frequency determinant and can go further to higher natural frequencies.

If $D_1 = D_2 = 0$, it degenerates to original frequency determinant, so it does not do any good. If $D_1 = D_2$, it likely means that first and second columns are the same. If this is the case, this method just fails.

In some cases, especially systems of many sections, the frequency determinant does not show parallelism. Experience shows that this method gives good results in most problems. However, this method also fails at sufficiently high frequencies.

2-6 Intermediate conditions - Branched system.

The effect of a joint point can be introduced into the calculation by means of a single point matrix.

This section is mainly for the derivation of the branch point matrix of the Modified Delta method and the Pestel-Mahrenholtz method.

The derivation of branch point matrix for the straight forward transfer method (and thus also for the Delta method) is fully explained in Ref. [3] and [4], but a brief discussion will be given here for the purpose of refreshing the reader's knowledge and moreover explaining the procedure for the M-D method and the P-M method.

A. Delta method.

Discontinuities are introduced in the forces whereas the main member deflection on each side of joint are the same, and identical to those of the branch at the same joint.

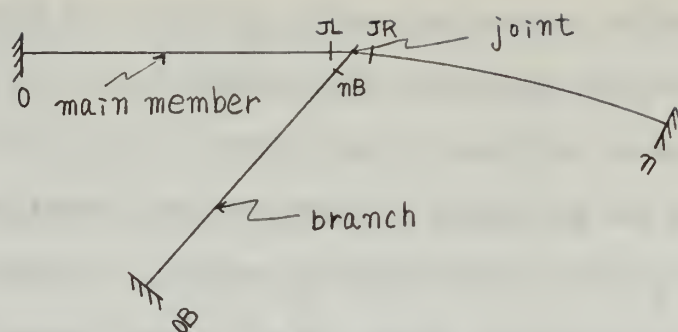


Fig. 2-6-1

By a suitable coordinate transformation, it is possible to express the force system of the branch in terms of the main system coordinates.

A free body diagram for the forces at the joint point is shown below.*

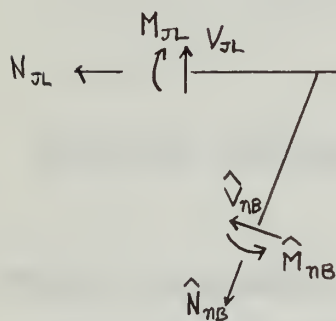


Fig. 2-2-2A

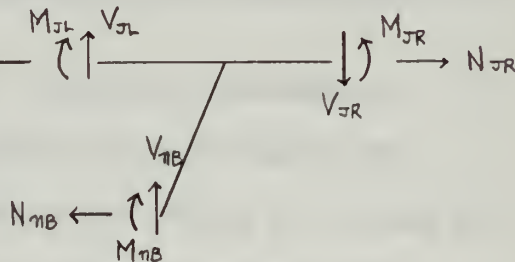


Fig. 2-2-2B

The forces at the joint in matrix form are.

$$\begin{bmatrix} M \\ V \\ N \end{bmatrix}_{JR} = \begin{bmatrix} M \\ V \\ N \end{bmatrix}_{JL} + \begin{bmatrix} M \\ V \\ N \end{bmatrix}_{nB} \quad (2-6-1)$$

The displacement is continuous at the joint, so Z_{JR} can be obtained by simply adding the force components of branch state vector to Z_{JL} .

The column matrix of nonzero component at the starting boundary will be symbolized by V_0 . Here $V_0 = \{1, 1, 1\}$

*Symbol \wedge indicates that the vector elements are expressed in terms of the branch coordinate system.

$$\hat{Z}_{\pi B} = \begin{bmatrix} \hat{d} \\ \hat{p} \end{bmatrix}_{\pi B} = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} \cdot V_0 \quad (2-6-2)$$

From equation 2-6-2

$$\hat{p}_{\pi B} = [R_2] \cdot [R_1]^{-1} \cdot \hat{d}_{\pi B} \quad (2-6-3)$$

The matrix product $[R_2] \cdot [R_1]^{-1}$ is called a spring matrix. To transform the coordinate system of the branch into main member coordinates at the joint, we use

$$\begin{aligned} \hat{d}_{\pi B} &= [G_1] \cdot d_{\pi B} \\ p_{\pi B} &= [G_2] \cdot \hat{p}_{\pi B} \end{aligned} \quad (2-6-4)$$

here; $G_1 = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $G_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix}$

where ϕ is a branch joining angle; refer to Appendix B-5.

then;

$$\begin{aligned} p_{\pi B} &= [G_2] \cdot [R_2] \cdot [R_1]^{-1} \cdot [G_1] \cdot d_{\pi B} \\ &= [S] \cdot d_{\pi B} \end{aligned} \quad (2-6-5)$$

$$([S] = [G_2] \cdot [R_2] \cdot [R_1]^{-1} \cdot [G_1]; \quad [S] \text{ is also called a spring matrix.})$$

Recalling the relation at the joint

$$\begin{aligned} d_{JR} &= d_{JL} = d_{\pi B} \\ p_{JR} &= p_{JL} + p_{\pi B} = p_{JL} + [S] \cdot d_{\pi B} \end{aligned}$$

we get Z_{JR} in matrix form

$$Z_{JR} = \begin{bmatrix} I & 0 \\ S & I \end{bmatrix} \cdot \begin{bmatrix} d \\ p \end{bmatrix}_{JL}$$

$$\text{or} \quad = [U_p] \cdot Z_{JL} \quad (2-6-6)$$

Here $[U_p]$ denotes the branch transfer point matrix.

B. Modified Delta method.

(In this section, the use of an asterisk * will denote a purged matrix or vector. Also note that the quantities d and p , with or without * and \wedge , represent 3×3 matrices whereas in the preceding subsection they represented 3×1 vectors.)

After we get the purging factors from $\Delta(\omega)$, the new starting matrix at location 0 is

$$[Z_0^1] = \begin{bmatrix} 0 \\ N^0 \end{bmatrix}, \quad [N^0] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -D_1 & -D_2 & 1 \end{bmatrix}$$

At the joint

$$[Z_{JR}^*] = \begin{bmatrix} d^* \\ p^* \end{bmatrix} \quad [Z_{JL}^*] = \begin{bmatrix} d \\ p \end{bmatrix}_{JL} \cdot [N^0]$$

For a branch, let the non-vanishing state matrix of the new starting state matrix $[Z_{0B}]$ be

$$[N_B] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -D_{1B} & -D_{2B} & 1 \end{bmatrix}$$

Here D_{1B} , D_{2B} are branch purging factors, the calculation of which will be explained later. At the joint

$$[\hat{Z}_{\pi B}^*] = \begin{bmatrix} \hat{d} \\ \hat{p} \end{bmatrix}_{\pi B} \cdot [N_B] = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} \cdot [N_B] = \begin{bmatrix} R_1^* \\ R_2^* \end{bmatrix}$$

$[R_1], [R_2], [R_1^*]$ & $[R_2^*]$ are defined by this expression.

Just as in equation 2-6-5,

$$\begin{aligned} [S^*] &= [G_2] \cdot [R_2^*] \cdot [R_1^*]^{-1} \cdot [G_1] \\ &= [G_2] \cdot [R_2] \cdot [N_B] \cdot [N_B]^{-1} \cdot [R_1]^{-1} \cdot [G_1] \\ &= [G_2] \cdot [R_2] \cdot [R_1]^{-1} \cdot [G_1] \\ &= [S] \end{aligned}$$

The spring matrix turns out to be the same as in the Delta method.

At the joint

$$\begin{aligned} p_{JR}^* &= [P_{JL}] \cdot [N_B^*] \cdot V_0 + [P_{JB}^*] \cdot V_0 \\ &= [P_{JL}^*] \cdot V_0 + [S] \cdot [d_{JL}] \cdot V_0 \end{aligned}$$

$[S]$ is independent of $[N_B]$, so p_{JB}^* is independent of $[N_B]$.

For Z_{JR}^* we have

$$Z_{JR}^* = \begin{bmatrix} I & 0 \\ S & 0 \end{bmatrix} \cdot Z_{JL}^* = [U_p] \cdot Z_{JL}^*$$

We see that branch transfer matrix $[U_p]$ is the same as in equation 2-6-6.

Purging factors can be obtained from $[R_1]$ or $[R_2]$. The phenomenon of columns approaching parallel could occur in $[R_1]$ and $[R_2]$; VIPIPE computes purging factors from $[R_1]$, because the writer was inclined to think that getting purging factors from $[R_1]$ would do more good in inversion of $[R_1]$. However, $[R_2]$ could have been used.

Purging factors D_{1B} and D_{2B} are obtained in the same way as in equation 2-5-1.

C. Pestel-Mahrenholtz method.

At the joint the equilibrium condition must be satisfied, and the deflections should be continuous.

$$P_{JR}^* = P_{JL}^* + P_{nB}^*$$

$$d_{JR}^* = d_{JL}^* = d_{nB}^*$$

At location 0B (boundary of a branch).

$$\hat{Z}_{0B}^{\circ} = \begin{bmatrix} 0 \\ N_{0B}^{\circ} \end{bmatrix} \cdot X_{0B}^{\circ}$$

Here, $[N_{0B}^{\circ}]$ is the nonvanishing state matrix of $[\hat{Z}_{0B}^{\circ}]$ and X_{0B}° is a branch correction vector,

let

$$[N_{0B}^{\circ}] = \begin{bmatrix} P_{0B}^{\circ} & 0 & 0 \\ P_{1B}^{\circ} & 1 & 0 \\ P_{2B}^{\circ} & 0 & 1 \end{bmatrix} \quad X_{0B}^{\circ} = \begin{bmatrix} X_{0B}^{\circ} \\ X_{1B}^{\circ} \\ X_{2B}^{\circ} \end{bmatrix}$$

Then, the state vector at location 0B for the built-in, in-plane case is

$$\hat{Z}_{0B}^{\circ} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ P_{0B}^{\circ} X_{0B}^{\circ} \\ P_{1B}^{\circ} X_{0B}^{\circ} \quad X_{1B}^{\circ} \\ P_{2B}^{\circ} X_{0B}^{\circ} \quad X_{2B}^{\circ} \end{bmatrix} \quad (2-6-7)$$

At the joint

$$Z_{JL}^* = \begin{bmatrix} d \\ P \end{bmatrix}_{JL} \cdot [N^{\circ}] \cdot X^{\circ} \quad Z_{JR}^* = \begin{bmatrix} d^* \\ P^* \end{bmatrix}_{JR} \cdot X^{\circ}$$

$$\hat{Z}_{nB}^* = \begin{bmatrix} \hat{d} \\ \hat{P} \end{bmatrix} \cdot [N_{0B}^{\circ}] \cdot X_{0B}^{\circ} = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} \cdot [N_{0B}^{\circ}] \cdot X_{0B}^{\circ} = \begin{bmatrix} R_1^* \\ R_2^* \end{bmatrix} \cdot X_{0B}^{\circ}$$

where $[R_1^*]$ and $[R_2^*]$ are

$$[R_1^*] = [R_1] \cdot [N_{OB}^0] \quad [R_2^*] = [R_2] \cdot [N_{OB}^0]$$

For $[N^0]$ and $[X^0]$, refer to section 2-4. Also $[R_1]$, $[R_2]$ are expressed in the same way as the preceding sub-section B.

$$\begin{aligned} [S^*] &= [G_2] \cdot [R_2^*] \cdot [R_1^*]^{-1} \cdot [G_1] \\ &= [G_2] \cdot [R_2] \cdot [N_{OB}^0] \cdot [N_{OB}^0]^{-1} \cdot [R_1]^{-1} \cdot [G_1] \\ &= [G_2] \cdot [R_2] \cdot [R_1]^{-1} \cdot [G_1] \\ &= [S] \end{aligned}$$

The spring matrix also turns out to be the same as in the Delta method.

From the requirement for continuous deflections at the joint we get

$$\begin{aligned} d_{\pi B}^* &= d_{JR}^* = d_{JL}^* \\ d_{\pi B}^* &= [G_1]^{-1} \cdot [R_1] \cdot [N_{OB}^0] \cdot X_{OB}^0 \\ &= [d_{JL}] \cdot [N^0] \cdot X^0 \end{aligned} \quad (2-6-8)$$

and

$$\begin{aligned} p_{\pi B}^* &= [S] \cdot d_{\pi B}^* = [S] \cdot [d_{JL}] \cdot [N^0] \cdot X^0 \\ p_{JR}^* &= [p_{JL}^*] \cdot X^0 + [S] \cdot [d_{JL}^*] \cdot X^0 \end{aligned}$$

For Z_{JR}^* we have

$$Z_{JR}^* = \begin{bmatrix} I & 0 \\ S & I \end{bmatrix} \cdot Z_{JL}^* = [U_p] Z_{JL}^*$$

We see that the branch transfer matrix $[U_p]$ is also the same as in equation 2-6-6. From equation 2-6-8, the branch correction vector is

$$X_{OB}^0 = [R_1^*] \cdot [G_1] \cdot [d_{JL}^*] \cdot X^0 \quad (2-6-9)$$

After we get the main member correction vector (X^0), we can get branch state correction factors in terms of X^0 values. As explained in section 2-4, iteration is performed in the same way.

2-7 Computer.

In FORTRAN 63, floating point quantities have an exponent and fractional part. In single precision arithmetic, the fractional part has 36 bits and the precision is approximately 10 decimal digits. In double precision arithmetic, the fractional part has 85 bits and the precision is approximately 25 decimal digits. In both cases the range of numbers is from 10^{-308} to 10^{308} .

In the FORTRAN 63 version of VIPIPE (as described later), either single precision (for conservation of computer time) or double precision (for greater computational accuracy) can be selected. In the FORTRAN 60 version, only single precision is available.

The natural frequencies of a planar piping system are found using program VIPIPE. Program VIPIPE has been tested with FORTRAN 60 and 63 using digital computer CDC 1604. The number of natural frequencies that may be found using VIPIPE and the accuracy, especially at high frequency, depends upon the floating point significant figure capacity. And the significant figure capacity required for any system is a function of the characteristics of the system itself.

CHAPTER III

DISCUSSION

3-1 Accuracy of methods and assurance of the solution.

Confidence in accuracy of methods was established by comparing frequencies of several systems for which these frequencies could be obtained by other means, i.e., by means of either a closed form analytical solution or by recourse to the Ref. [5] .

The accuracy is very good for the lower natural frequencies in the values got using the Delta method, the Modified Delta method, and the Pestel-Mahrenholtz's method.

In most cases, up to fourth mode frequencies, the difference between the system natural frequencies obtained by analytical means and those obtained using VIPIPE are less than 0.004%. As frequency increases we got more accurate natural frequency values from the M-D method and the P-M method comparing to the original transfer method (Delta method).

In the sample problems reported in Appendix E, we have found that for straight, simple configurations (where the phenomenon of the columns becoming parallel is pronounced) the P-M and M-D methods give one or two more natural frequencies than can be obtained by the Delta method or the original transfer method. However, in more complicated configurations it seems to be possible to get three or four more natural frequencies. In the simpler cases, there are comparison values available from "exact" theory so that the accuracy of the results may be established. However, in the more complicated cases, such comparison values are not available. However, we have reasonable assurance of the integrity

of these results since we have been able to get the same values from the M-D method and from several different P-M subprocedures. Thus, using these methods, we seem to be able to double or triple the frequency range in which valid results may be obtained.

Details of the systems analyzed and the results obtained may be found in Appendix E.

3-2 Limitations and some suggestions.

Non-stiff hangers may be introduced into the system by a point matrix with appreciable linear and/or torsional spring compliances. But, for stiff hangers the transfer method fails, so we cannot use program VIPIPE for natural frequencies of a system which has stiff hangers. In particular, the suggestion of Fink [4] that rigid hangers can be successfully handled by use of idealized very stiff exterior springs proves to be untenable.

Marguerre and Uhrig suggested possible way to overcome the difficulty relating to stiff hangers by introducing unknowns [2] . Introducing unknowns means method abandoning the transfer concept, so we cannot use VIPIPE directly to solve such a problem. However, the writer thinks that there is a possibility of modifying VIPIPE to solve such problems by getting state quantities at each node and assuming values for the unknowns. For details refer to reference [2] .

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APPENDIX A

DESCRIPTION OF PROGRAM

A-1 General remarks

Program VIPIPE is a CDC FORTRAN 63 language digital computer program designed for use in the vibration analysis of planar piping systems, originally developed by George E. Fink [4] . The program VIPIPE is modified and augmented by the writer to use Delta method (same results as original transfer method, but with less amount of calculation), Modified Delta method, and Pestel and Mahrenholtz's remainder method with double precision or single precision arithmetic.

In- and out-of-plane undamped natural frequencies may be found through the use of program VIPIPE. How this program may be used is fully explained in Appendix B. Considerable flexibility with regard to the mathematical model employed is provided in the program. For instance, one may use combination of a distributed mass treatment for straight section with a lumped mass treatment for curved sections (this method gives the greatest accuracy coupled with least machine time) or a lumped mass for all sections. Also one may consider or neglect the effect of shear deflection and rotational inertia, singly or together.

Codes for the boundary conditions and various methods are interpreted within the program.

Problem solutions proceed by iterative processes with radian frequency the independent variable. The sign of the frequency determinant or remainder is used to control the search for and convergence to a system natural frequency.

Output and transfer to the next method/or transfer to the next problem or to the end is provided in the program when it reaches significant figure limitation or previously set iteration limit, and also when a method fails.

All subroutines are provided in binary deck in single precision and double precision arithmetic in the FORTRAN 63 version. Modifications of the program may be accomplished by changing FORTRAN cards only in the main program; the subroutines are in binary to conserve space and time.

A-2 Program structure

The program consists of a six section main body and twenty-seven subroutines. In the FORTRAN 63 library, GRAPH subroutine is not available. So GRAPH subroutine is provided in binary, and built into the program. Also, in the FORTRAN 63 library, hyperbolic functions are not available, so these functions are computed in exponential form.

The function of each section of the main body and of each subroutine is given below.

A. Main body sections and their functions.

INPUT Controls read-in of data, allocates storage locations for arrays and subscripted variables, decides the first method to perform.

INVARIANTS Computes and assigns appropriate subscripts to such system component characteristics as moment of inertia and radius of gyration, computes frequency increment for the first two modes, and sets the solution acceptability criterion.

**CONTROL
IN-PLANE**

Constructs transfer matrix of the system for the in-plane case by calling for various subroutines in a sequence governed by the system geometry, the mathematical model specified, and the desired assumptions.

**CONTROL
OUT-OF-PLANE**

Performs the same function as CONTROL IN-PLANE but for the out-of-plane case.

ITERATION

Evaluates the frequency determinant (modified frequency determinant in the M-D method) or the remainder and utilizes its sign in controlling the iterative search for mode frequencies. Initiates output when the specified frequency range has been fully explored, the required number of modes found, the specified iteration number has been reached, or the significant figure limit of computer reached.

OUTPUT

Controls output format, provides graph output, seeks the method to perform.

B. Subroutines and their functions.

SUBSEC

Subdivides components to be treated by lumping mass on the basis of L/D ratio and starting frequency or required mode numbers. (L/D ratio for curved section means ratio of radius of curvature to diameter and included angle of arc, as applicable.)

MATMUL

Constructs the transfer matrix of those components treated by lumping mass for only one point mass and massless subsection.

FINMAT

Constructs the system state matrix (6x3) by successive premultiplication of the partial system state matrix

by the transfer matrix of each component as soon as each of the latter is constructed. For the lumped mass system, gives successive premultiplication by the transfer matrix constructed by MATMUL as many times as the number of lumped masses.

FINBRA

Constructs the state matrix of a branch system by successive pre-multiplication of the partial branch state matrix by the transfer matrix of each component of a branch as soon as each of the latter is constructed. For the lumped mass system, gives successive pre-multiplication by the transfer matrix constructed by MATMUL as many times as the number of lumped masses.

STATEM

Constructs starting state matrix from the starting boundary condition of main member, for each method.

STATEB

Constructs starting state matrix of a branch system from the far end boundary condition of a branch for each method. Computes the purging factors of the branch in the M-D method.

BRCOR

Constructs branch correction matrix for the assumed branch starting state matrix at a certain frequency in the P-M method. Constructs branch purging matrix in the M-D method.

DELMA

Constructs frequency determinant in the Delta method. Evaluates the purging factors from the frequency determinant of the Delta method and constructs the modified frequency determinant after another round of calculation in the M-D method. For the P-M

method, computes the correction factors for each sub-procedure. For each sub-procedure, if it fails by the fact that denominator is zero, the rows are changed and a second test is made. It also provides transferring to the next method or sub-procedure if it fails.

DISTM, DISTMO, SFIELD, SFIELO, CFIELD, CFIELO, POINT, POINO, RIGID, RIGIO, STIFCO, HANGER, HANGE0, BRANCH, BRANCO, STAVEC, STAVEO, INVERT.

For the above subroutines refer to reference [4] .

A-3 Program nomenclature.

B1,B2,B3	Correction factors for assumed state quantities of the far end of a branch.
BC	Discriminant. Controls boundary correction in the P-M and M-D method.
BL	Number of the last iteration.
D1,D2	Purging factors in the M-D method. Correction factors in the P-M method.
DEL	Array of numerical values of frequency determinants and array of remainders.
DV	Numerical value of a determinant in denominator of X1 and X2.
EDL	Array of numerical values of frequency determinants or remainders in single precision for input to GRAPH.
ERM	A discriminant same as REM in the problem statement data card.
FF	Discriminant. Controls output to provide a print

output that analysis is terminated due to significant figure limitation in the P-M method.

FINK Discriminant. Controls constructing the frequency determinant or modified frequency determinant in the M-D method.

HH Discriminant. Controls transferring to another sub-procedure or to another way of testing in case one fails in the P-M method.

HMH Discriminant. Controls subsectioning in lumped mass system.

K Discriminant. Controls grid in the GRAPH output.

KA Discriminant. Terminates iteration when specified iteration number is finished.

KKK Discriminant. Controls transferring to the output in case the last sub-procedure in the P-M method fails or controls transferring to another three sub-procedures (D,E,F) after three sub-procedures (A,B,C) are finished in case subprocedure C fails in Selection 3 (for Selection, see Appendix B-9-6).

MAT Discriminant. Controls matrix multiplication in the lumped mass system.

NUMPTS Number of points in one graph (except the last).

NUMEND Number of points in the last graph.

P1, P2 Assumed starting state quantities of the main member.

PMM Discriminant. Controls three methods (Delta, M-D, P-M method) specified.

PPM Discriminant. Assures that same procedures are used for out-of-plane solution that were used for in-plane solution.

PM Discriminant. Controls three methods to be used.

PP Discriminant. Controls the three sub-procedures (A,B,C) in the P-M method.

RR Discriminant. Controls testing by interchanging two rows in case one sub-procedure fails because the denominator of X1 or/and X2 is zero in the P-M method.

R3 Array name. Matrix product $[R_1]^{-1} \cdot [G_1]$, used in the branch correction.

REM Discriminant. Controls three ways of selecting sub-procedures in the P-M method.

Q1,Q2,Q3 Assumed starting state quantities of a branch.

SS Discriminant. Controls the three sub-procedures (D, E,F) in the P-M method.

XX Array name. Branch correction matrix (3x3) in the P-M method and branch purging matrix in the M-D method.

X1,X2,Y1,Y2 Correction factors for assumed starting state quantities of the main member in the P-M method.

YY Array name. Branch correction matrix (3x3x23).

For other nomenclature see reference [4].

APPENDIX B

INSTRUCTION FOR PROGRAM USE

B-1 General remarks.

Appendix B is intended to provide complete instructions for using program VIPIPE. VIPIPE is FORTRAN 63 language program with twenty-seven subroutines. All subroutines are provided in a binary deck so that the compiling time is reduced as is the volume of the deck.

Program VIPIPE can be used in single precision or double precision arithmetic. Also VIPIPE is provided in FORTRAN 60 language with single precision.

The Delta method, the M-D method, and the P-M method are all incorporated in program VIPIPE. The M-D method and the P-M method do appear to permit obtaining more natural frequencies (one, two, or possibly several more) than can be obtained with the Delta method (same result as the original transfer method). However, the P-M method has the disadvantage of introducing an odd behavior of the remainder. In some cases, this odd behavior can easily be misinterpreted so as to lead to false frequency determination. However, there are a whole group of associated P-M modifications* and the odd behaviors occur at different frequencies for the different modifications. Thus, by making the calculation more than once, using a different modification each time, the true natural frequencies can be sorted out.

Program VIPIPE uses the Delta method as the basic procedure. If more frequencies are required than have been obtained by this method,

*P-M modifications means sub-procedures of the P-M method.

one may then use one or more of the available associated P-M modifications and/or the M-D method. Graphical output will assist in identifying the natural frequencies. The writer recommends to use the P-M modifications as original method in getting more natural frequencies, and to use the M-D method for purposes of comparison.

The accuracy and number of natural frequencies that can be obtained using the methods depends on the capacity of handling numbers accurately. As we can see in Appendix E, double precision arithmetic gives better results especially for complicated systems for which many matrix multiplication are performed. But using double precision arithmetic results in considerable increase of machine time. And also for various methods, the machine time is different. The Delta method and the P-M method each take about the same amount of machine time, but the M-D method takes almost twice this time.

So a user should consider the frequency range under investigation and the nature or characteristics of the system as well as the machine time.

For comparison purposes Section B-9-6 of this Appendix lists machine times for one iteration for some example problems using each of three methods with double and single precision arithmetic.

Complicated systems are divided into one main member and branches. System components are defined as sections of pipe, straight or of constant curvature, an elbow, a spring hanger, a coupling, a flange. Diameters, wall thickness, density, elastic and shear modulus may vary from component to component, but may not vary within a component. Both torsional and linear spring rates (for hangers) must have finite values. Thus perfectly rigid hangers cannot be treated.

B-2 Coordinate system.

A local coordinate system is constructed at each point of interest in the system. A right handed coordinate system is used whose X axis is the local tangent to the central axis of the main member, always directed away from the left end. The X-Y plane coincides with the quiescent plane of system. The Z axis is always directed downward. The direction of the local Y axis is chosen so as to complete the right handed system.

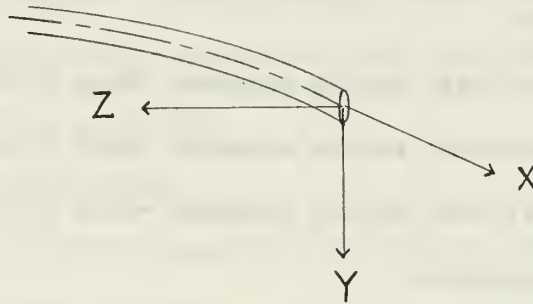


Fig. B-2-1

The branch coordinate starts from the end of the branch farthest from the joint point with the main member. The plane of the branch coincides with the X-Y plane. The positive Z axis of the branch and main member are parallel and have the same sense. The Y axis is oriented so as to complete a right handed system.

B-3 Data required.

Extensive properties.

- a. outside diameter (D) - inches.
- b. wall thickness (DI) - inches
- c. radius of curvature (RHO) - inches (Negative, if center of curvature lies on negative Y axis).

- d. length (TL) - inches (for straight section only; for curved sections, length is computed by the program).
- e. arc central angle (THETA) - degrees
- f. linear spring constant in X direction (CLX) - pounds per inch
- g. linear spring constant in Y direction (CLY) - pounds per inch
- h. linear spring constant in Z direction (CLZ) - pounds per inch
- i. torsional spring constant about X axis (CTX) - in-lb/radian
- j. torsional spring constant about Y axis (CTY) - in-lb/radian
- k. torsional spring constant about Z axis (CTZ) - in-lb/radian

Intensive properties

- a. density of component (AAMU) - lb/ft³
- b. elastic modulus (AE) - psi
- c. shear modulus (AG) - psi

B-4 Component identification.*

Components are identified by two numbers.

A. All components have a component sequence number starting from the most left component of the main member. This sequence number is identified by the order of data cards containing component extensive and intensive properties. This can be understood from the figure B-4-1.

*Also see pp. B-5 to B-7 of reference [4] .

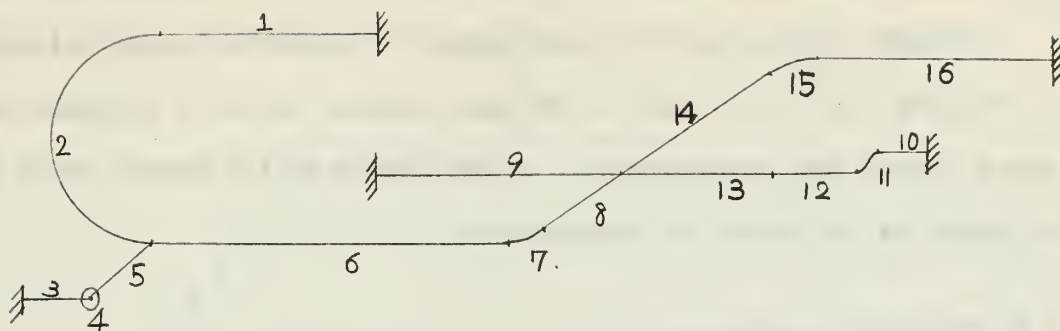


Fig. B-4-1

B. A branch component identification number (NNBR) is read in as data. It indicates whether it is in a branch or main member, if it is in branch, the position in branch.

- NNBR 0 main member
- NNBR 1 the first component of a branch (farthest from the intersection)
- NNBR 2 intermediate component of a branch
- NNBR 3 the last component of a branch
- NNBR -1 the only component of a branch

This can be understood from the figure B-4-2

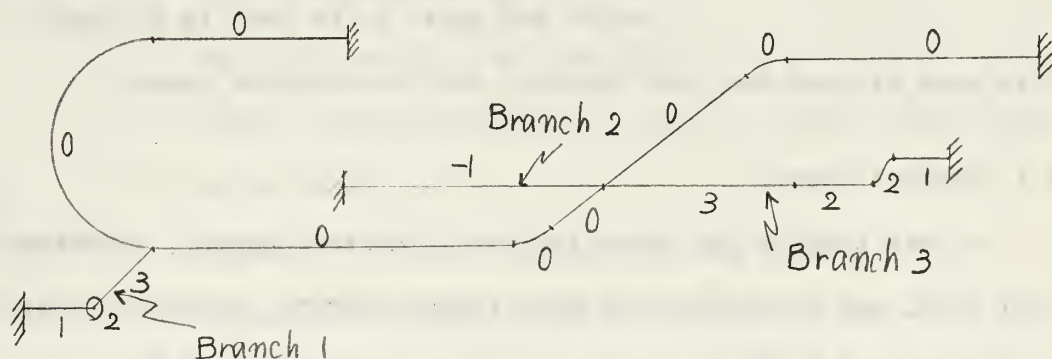


Fig. B-4-2

B-5 Branch joining angle

Branch joining angle to main member is measured counter-clockwise in degrees from the X axis of the main member, as it is oriented immediately before the inter-section, to the X axis of the branch as it is oriented at the point of intersection.

B-6 Boundary conditions*

Five common boundary conditions, those of a fixed, free, pinned, propped, or a roller supported end are indicated by the code given.

The program interprets the code and forms the required state matrix (6x3).

Boundary condition code:

- a. SV 1 fixed end
- b. SV 2 free end
- c. SV 3 pinned end
- d. SV 4 propped end
- e. SV 5 roller supported end
- f. SV 0 other than one of the above.

(The user must construct the necessary state vector and cause to be read in as data).

This code is used for both in-plane and out-of-plane cases.

B-7 Control cards.

A user sets up for execution with a master control, a FORTRAN control card, and combination of END, FINIS, EXECUTE, transfer cards, binary cards, and FORTRAN cards properly placed in FORTRAN 63 deck (JOB and END cards are used in FORTRAN 60). Figure B-7-1 shows the deck assembly

*For further details, see pp. B-8 to B-10 of reference [4].

for FORTRAN 63 use.

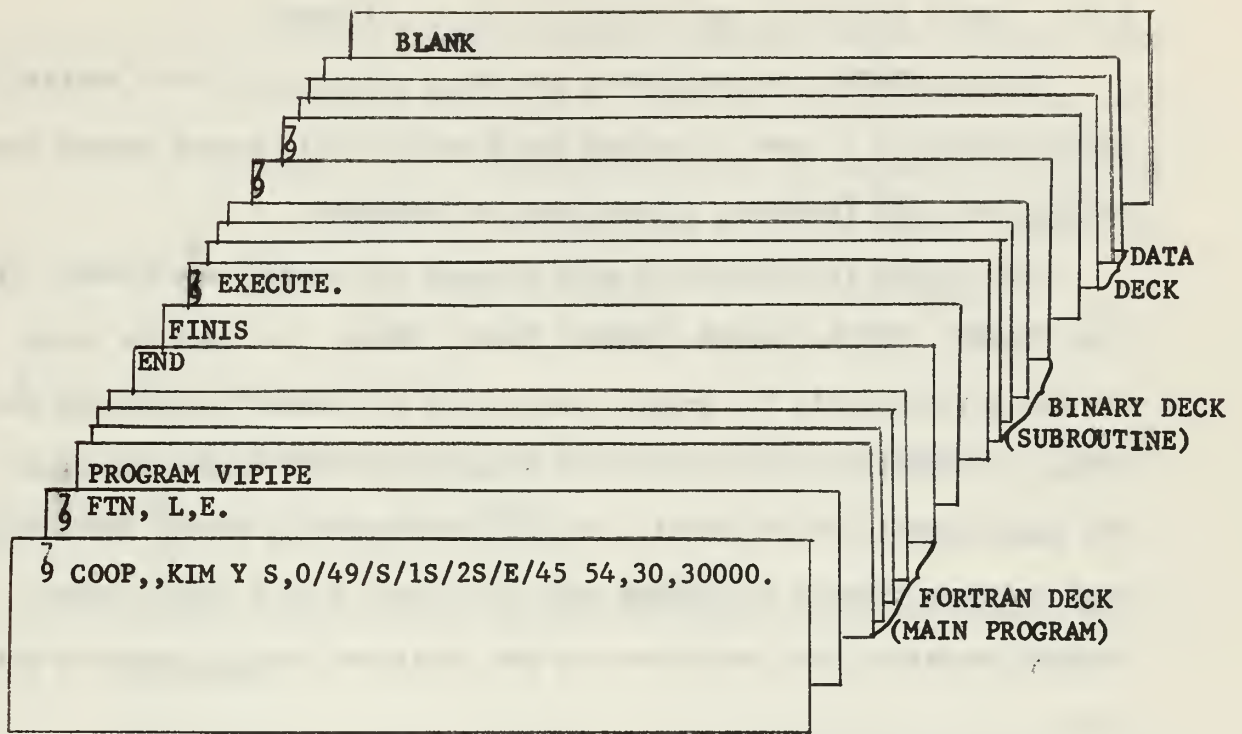


Fig. B-7-1

B-8 Data deck assembly.

The data deck is ordered as follows:

- a. problem statement card.
- b. boundary condition data card(s).
- c. branch joining angle data card(s). (deleted when system has no branches)
- d. component extensive properties data card(s).
- e. component intensive properties data card(s).

The user may obtain solutions for any number of problems desired; in this case each problem data deck is assembled just behind the other.

Following the last problem deck a blank card should be added.

B-9 Flexibility of program VIPIPE.

B-9-1 Double precision and single precision arithmetic.

Program VIPIPE can use double precision process only with FORTRAN 63. Double precision is used in system state matrix (UU), branch system state matrix (VV), and frequency determinant or remainder.

Type double is declared in main program and subroutines FINMAT, FINBRA, STATEM, STATEB, BRANCH, BRANCO, BRCOR, DELMA. By removing cards declaring type double the program can easily be changed into single precision. Subroutines are provided in binary deck with single precision and double precision arithmetic, so with corresponding binary subroutine deck and main program in FORTRAN with or without a card (card number 000120) declaring type double we can use single or double precision arithmetic.

B-9-2 GRAPH.

A user can get GRAPH output by option. The values of the frequency determinant and the values of the remainder are not usually of the same order of magnitude throughout the frequency range. Moreover, because of the infinite phenomenon of the P-M method, it is not generally useful or convenient to get one graph for the whole frequency range.

1. Number of points in one graph can be changed by changing NUMPTS in card 006090.
2. Can get graph with grid or without grid by changing card 006100.

K = 1 with grid

K = 0 without grid.

3. Graphs are labeled from 1,2,3,...up to 10. Graphs beyond tenth one will be labeled as 10. The last graph is labeled LAST.
4. For each graph, the whole frequency range is moved (except the first one) so that the first point of each graph starts from 0. The corresponding value of frequency determinant (or remainder) for the first point and NUMPTS (NUMEND for the last graph) are printed in the print output.

B-9-3 Shear area form factor.

A shear area form factor of one half is built in program.* This may be changed by changing card 000600.

B-9-4 Starting frequency, frequency increment and solution acceptability criterion.

Using the M-D or the P-M method, we are expecting to get more natural frequencies at high frequency. Thus choosing the proper starting frequency for the M-D or the P-M method, following the Delta method can save machine time. This can be done by putting the wanted starting frequency in problem statement data card.

To compute the starting frequency increment, the program constructs a synthetic straight pipe, equal in length to the length of the main member, and having diameter, wall thickness, and intensive properties equal in magnitude to a weighted average of those properties of the composite members. One quarter of the fundamental frequency of this synthetic pipe is taken as the starting frequency increment (card 001390). And one thousandth of this increment is taken acceptability criterion (card 001410). When the iteration reaches this solution acceptability

*See reference [7] .

criterion, the corresponding natural frequency is determined using linear interpolation.

This frequency increment is for the first two mode frequencies, but the acceptability criterion is the same for all mode frequencies. After second mode frequency is found, frequency increment is one tenth of the difference between the last two mode frequencies found (card 003950).

These can be altered by changing cards. The writer found that a smaller frequency increment gives better results especially at high frequency with the P-M method.

B-9-5 Subsectioning.

Subsectioning for the lumped mass treatment of components is carried on in subroutine SUBSEC. A component is divided into maximum of twelve subsections.

For all components having a length to diameter ratio greater than six, the number of subsections is computed as two times the value of a parameter HM which appears in SUBSEC (HM is number of modes to be sought). If HM is greater than six, HM is given the value of six. In case the starting frequency is greater than 800 radians per second, HM is also given the value of six. HM can be controlled by a parameter HMH in the main program (card 001250). If HMH is zero, HMH does not affect the value of HM. By changing HMH to other than zero we can change the value of HM equal to that of HMH.

Poor subsectioning might give some loss of accuracy in determining natural frequencies, especially at high frequency. Checking the natural frequencies by increasing the number of subsections using parameter HMH is recommended.

B-9-6 Machine time and printing line limit.

In FORTRAN 63, estimated time is set to thirty minutes and printing line limit is set at 30000 lines in VIPIPE. These limits can be altered by changing the master control card. If a limit is reached, computation stops automatically.

For reference, machine time for one iteration of some example problems is given below.

	Example System*	Delta meth.	M-D meth.	P-M meth.
Double Prec.	system 1	0.131 sec.	0.247 sec.	0.144 sec.
	system 4	2.10 sec.	4.22 sec.	2.14 sec.
	system 5	4.799 sec.	6.90 sec.	3.505 sec.
Single Prec.	system 1	0.044 sec.	0.117 sec.	0.045 sec.
	system 5	0.377 sec.	0.968 sec.	0.49 sec.

B-9-7 Performing various methods.

One selects the methods to be employed by use of the problem statement data card. Then the program picks the first method in the INPUT section of the main program with a discriminant PM, in the order: Delta method, M-D method, P-M method.

Finishing a method, the OUTPUT section of the main program opens to the next method. Finishing all specified methods, the program goes to the out-of-plane portion, to the next problem, or to the end.

In the P-M method, there are six sub-procedures. These sub-procedures are controlled by discriminatns PP, SS.

PP : 0.	subprocedure A,	SS : 0.	sub-procedure D,
PP : 1.	sub-procedure B,	SS : 1.	sub-procedure E,
PP : 2.	sub-procedure C,	SS : 2.	sub-procedure F.

*For example systems refer to Appendix E.

Each sub-procedure has another possible way by interchanging rows of final determinant (refer to section 2-4). If one fails, the alternative one is tested. If one succeeds, then go to the next sub-procedure. There are three ways of selecting sub-procedures by discriminant REM.

REM: 2. three sub-procedures in the order: A,B,C. - Selection 1

REM: 1. three sub-procedures in the order: D,E,F. - Selection 2

REM: 0. six sub-procedures in the order: A,B,C,D,E,F. - Selection 3

If one wants just one sub-procedure, by changing PP to a value of 2 (card 000710) in selection 1 only sub-procedure C is used, or changing SS to a value of 2 (card 001000) in selection 2 only sub-procedure F is used. If one wants just two sub-procedures, by changing PP or SS to a value of 1 sub-procedure B and C or sub-procedure E and F, respectively, are used.

B-9-8 Limiting the number of iterations.

One can limit the number of iterations, for each problem, by a parameter HL in the problem statement data card. In the P-M method, because of the infinite phenomenon, the frequency increment in iteration can become very small. Thus the iteration might keep going on indefinitely.

The program permits six hundred iterations as maximum. The writer found three hundred iterations to be adequate to get nine natural frequencies with the acceptability criterion set in the program.

B-9-9 End conditions.

A boundary condition state vector must have three non-zero and three zero elements. Frequently, however, an actual condition may not meet this specification. For instance, an end may be supported or connected to a piece of machinery. Then we approximate it as being held in a

mounting block having specified linear and torsional spring rates, and to this attach a fictitious massless short section.

Through the use of this sort of fictitious end, a proper end condition state vector may be constructed for any real situation with little or no loss of accuracy.

For hangers exhibiting coupling, systems having unusual flexibility, and flanges and couplings refer to pp. B-19 to B-20 of reference [4] .

B-10 Format of data

B-10-1 Problem statement

The card has 15 fields under control of the following field specifications.

312, 4F3.0, 2F8.0, 4F2.0, 2F4.0

Field names and the information each field conveys are given below in order.

A. Field names and content.

1. NS number of components.
2. NBR number of branches.
3. IOP code for in-and/or out-of-plane solutions.
4. HM number of modes sought.
5. AMYK code calling for the use of lumped or distributed mass model
6. SD code calling for shear deflection inclusion or neglect
7. RI code calling for rotational inertia effect inclusion or neglect
8. OMEGAG upper limit of frequency range of investigation
9. OMEGAI lower limit of frequency range of investigation and initial iteration frequency. If 0 is placed, it will begin at .01 radians per sec.

10. GDABC code calling for print output..
11. BRC code calling for the inclusion or exclusion of branch correction factors in P-M method and purging factors in M-D method.
12. GRAPH code specifying whether there is to be graph output.
13. REM code for choosing methods in P-M method.
14. PMM code calling for the combination of 3 methods of Delta, M-D, P-M method which a user wants.
15. HL number of iterations.

B. Codes used.

1. IOP 0 Both in-and out-of-plane mode frequency sought (in this case, in-plane frequencies are sought first).
2. IOP 1 In-plane frequencies are sought.
3. IOP 2 Out-of-plane frequencies sought.
4. AMYK 1. Lumped mass model,
5. AMYK 0. Distributed mass model,
6. SD 0. Consider shear deflection.
7. SD 1. Neglect shear deflection.
8. RI 0. Consider rotational inertia.
9. RI 1. Neglect rotational inertia.
10. GDABC 0. Print solution only.
11. GDABC 1. Print solution and input data.
12. GDABC 2. Print solution and input data and graph data.
13. BRC 0. Do not consider branch correction factors or purging factors.
14. BRC 1. Consider the branch correction factors or purging factors.
15. GRAPH 0. No graph output.
16. GRAPH 1. Graph output.

17. REM 0. All six sub-procedures in the P-M method (A,B,C,D,E,F).
18. REM 1. Three sub-procedures in the P-M method (D,E,F).
19. REM 2. Three sub-procedures in the P-M method (A,B,C).
20. PMM 1. Delta method only.
21. PMM 10. M-D method only.
22. PMM 100. P-M method only.
23. PMM 11. Delta, M-D method.
24. PMM 101. Delta, P-M method.
25. PMM 110. M-D, P-M method.
26. PMM 111. Delta, M-D, P-M method.

B-10-2 Boundary conditions.

Field specification 22F7.3

Boundary conditions are read in separately for in- and out-of-plane cases. When both cases are to be investigated, in-plane case data are read in first.

$N + 2$ fields must be filled in (N is number of branches). The first and last fields are first and last boundary condition codes of the main member. Branch boundary conditions are entered in the intermediate fields in accordance with the branch sequence number. When a boundary condition is not describable by one of the codes, a zero must be entered in its data field. For each zero an additional data card is required. The limitation in non-zero state quantities at the boundary must be three (refer to B-9-9).

B-10-3 Branch joining angle (degrees).

Field specification is: 20F7.3

B-10-4 Component extensive properties and branch component identification.

A data card for each system component, ordered by sequence number.

Field specification is: 3F5.2, 2F6.2, 6F7.2, I2

Field names are: in order,

D, DI, RHO, TL, THETA, CLX, CLY, CLZ, CTX, CTY, CTZ, NNBR

B-10-5 Intensive properties.

The first intensive properties data card has four fields with specification:

F7.3, 2E10.2, F2.0

Field names are: AAMU, AE, AG, DD*

*When the intensive properties of all components are the same, the DD field is left blank. When the intensive properties are different, the DD field is non-zero, then an intensive data card must be prepared for each of the remaining components (including hangers). Additional data cards are identical to the first one except that the DD field is blank.

APPENDIX C

FLOW DIAGRAMS

C-1 General remarks.

Flow diagrams are included for all parts of the main program, and for several subroutines. Since CONTROL OUT OF PLANE, FINBRA flow diagrams are identical in structure to those of CONTROL IN PLANE and FINMAT respectively, only the latter are included. Reader can refer to the flow diagram of STAVEC and STAVEO of reference [4]. Since the flow diagram structures of SUBSEC, BRANCH and BRANCO are similar to those of reference [4], they are not included here.

Flow diagrams of other subroutines that are not included in this paper are relatively simple ones, so the reader can refer directly to the program.

C-2 Index of flow diagram.

Main program.

Flow Diagram.	Page.
INPUT	63
INVARIANTS	65
CONTROL IN PLANE	66
ITERATION	68
OUTPUT	70

Subroutines

FINMAT	73
STATEM	74
STATEB	75
BRCOR	76
DELMA	77

C-3 Flow Diagrams

Flow diagrams are used to show the sequence of steps in a process. They are often used in business and industry to show the flow of materials, information, or money. Flow diagrams can be used to identify inefficiencies and improve processes. They can also be used to train new employees and to document processes for future reference.

Flow diagrams are typically created using a set of standard symbols. These symbols represent different types of steps in a process, such as starting points, decision points, and end points. Arrows are used to connect the symbols and show the flow of the process.

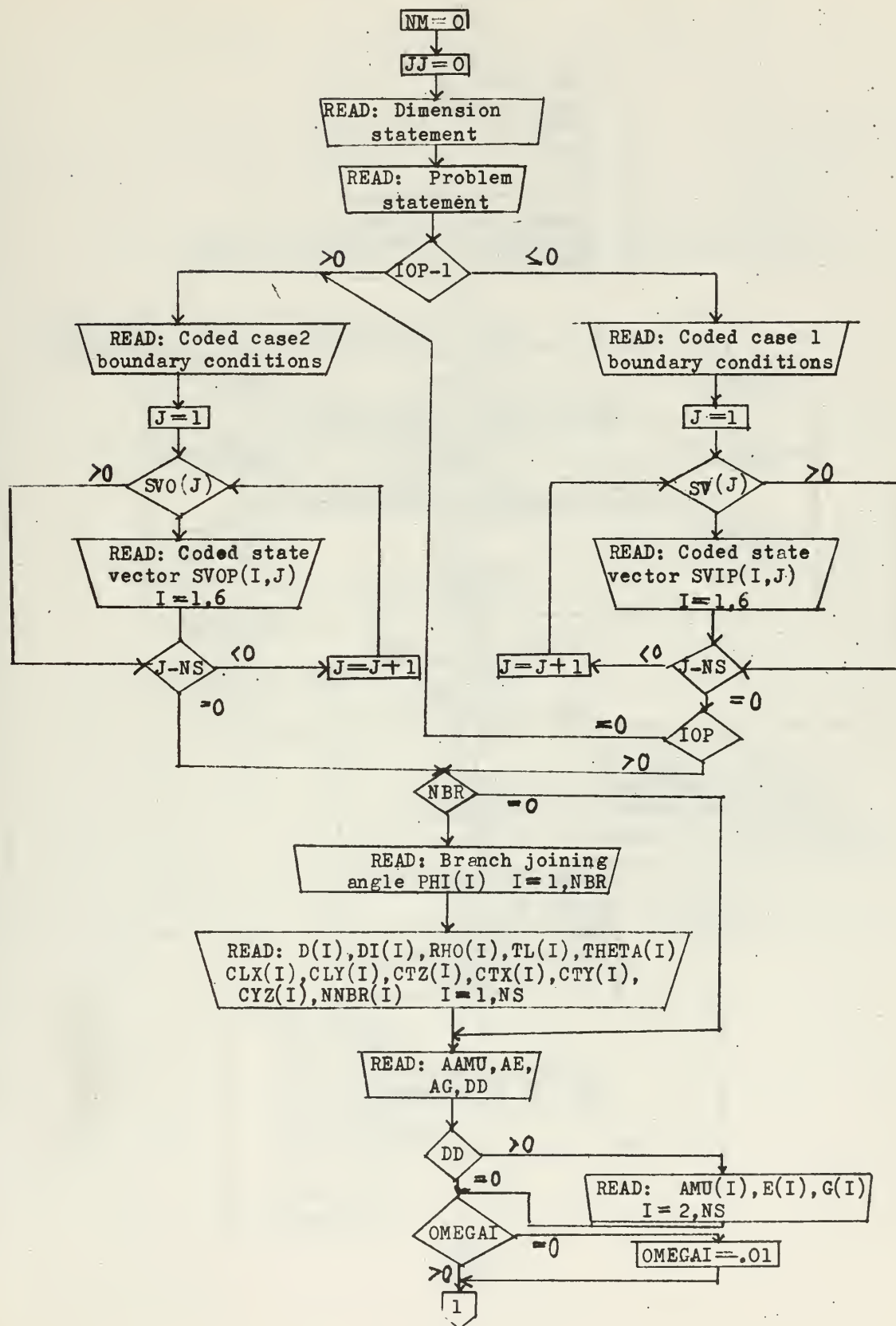
Flow diagrams can be used in a variety of ways. They can be used to show the flow of materials in a manufacturing process, the flow of information in a business process, or the flow of money in a financial process. They can also be used to show the flow of a project or a task.

Flow diagrams are a useful tool for understanding and improving processes. They can be used in a variety of ways and are often used in business and industry. Flow diagrams can help to identify inefficiencies and improve processes, and they can also be used to train new employees and to document processes for future reference.

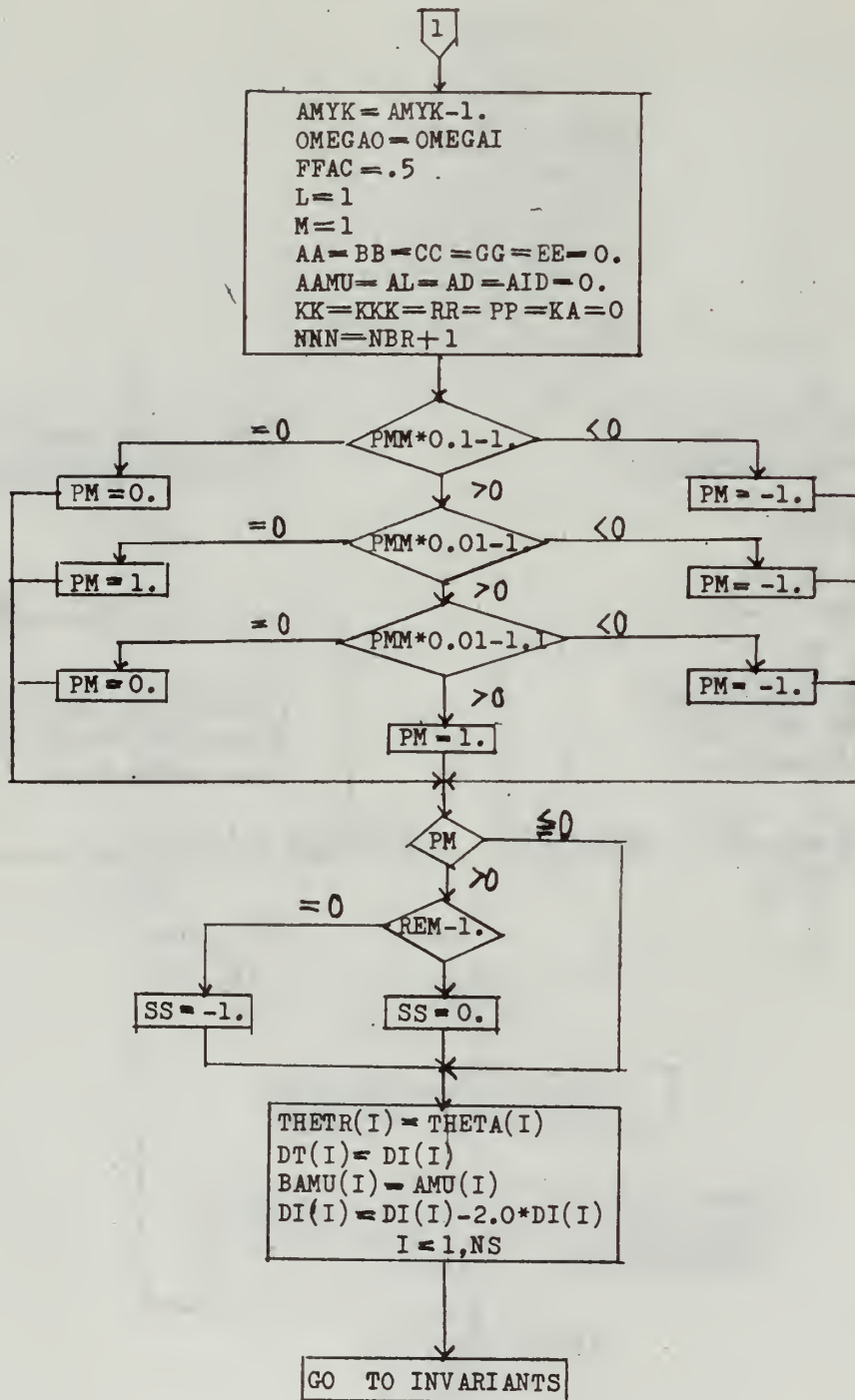
Flow diagrams are typically created using a set of standard symbols. These symbols represent different types of steps in a process, such as starting points, decision points, and end points. Arrows are used to connect the symbols and show the flow of the process.

Flow diagrams can be used in a variety of ways. They can be used to show the flow of materials in a manufacturing process, the flow of information in a business process, or the flow of money in a financial process. They can also be used to show the flow of a project or a task.

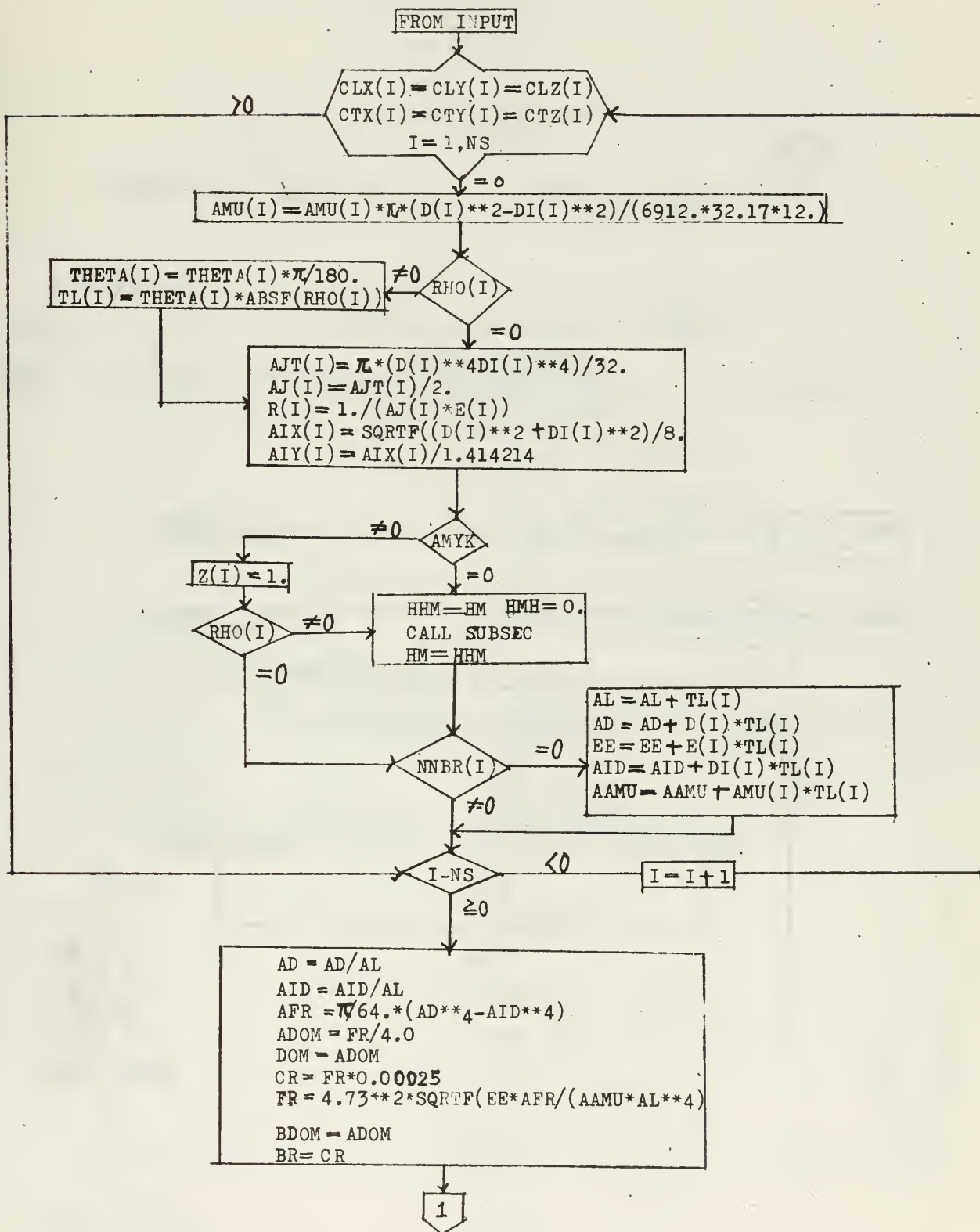
INPUT FLOW DIAGRAM



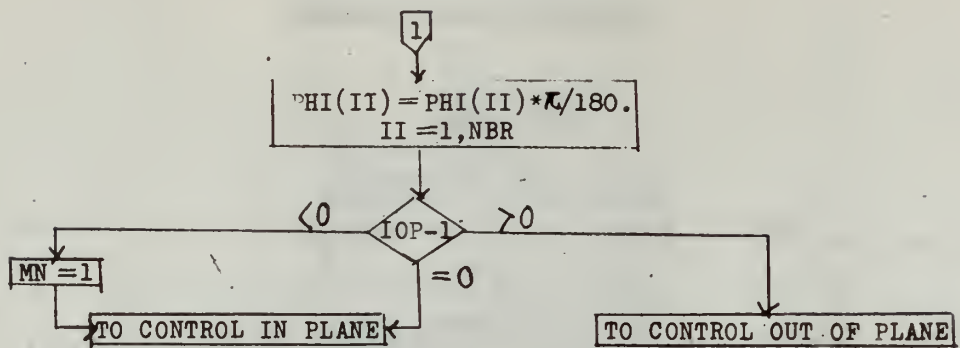
INPUT FLOW DIAGRAM(CONT.)



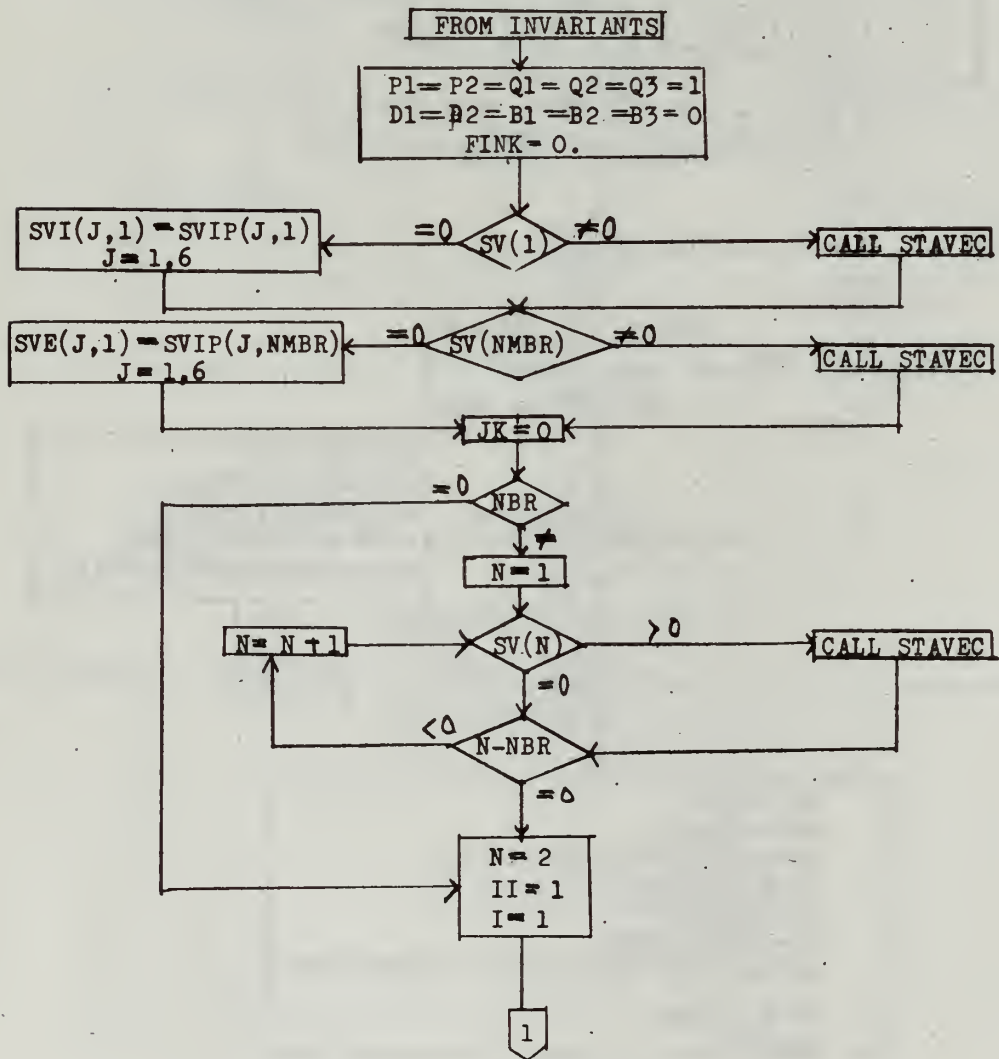
INVARIANTS FLOW DIAGRAM



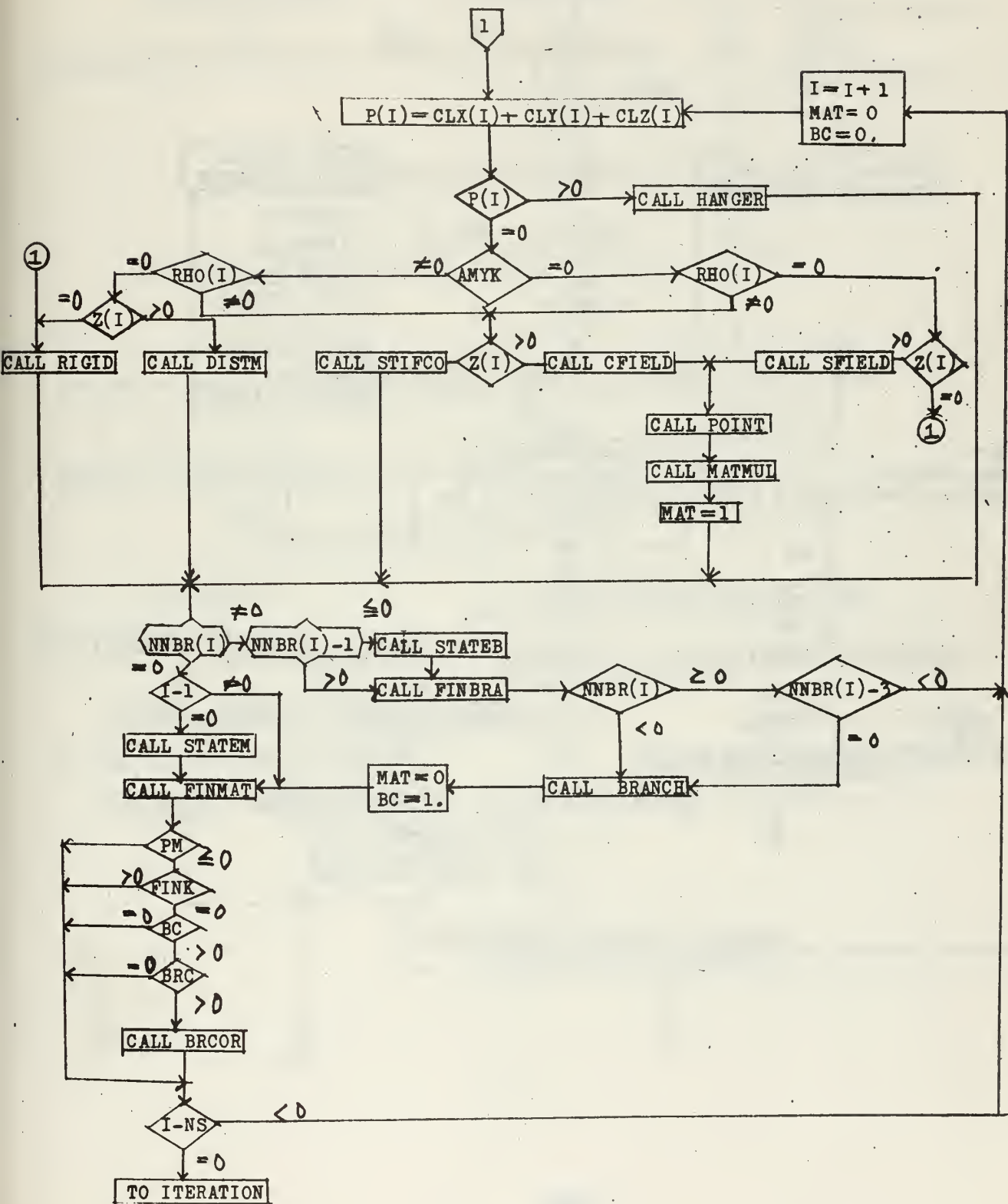
INVARIANTS FLOW DIAGRAM(CONT.)



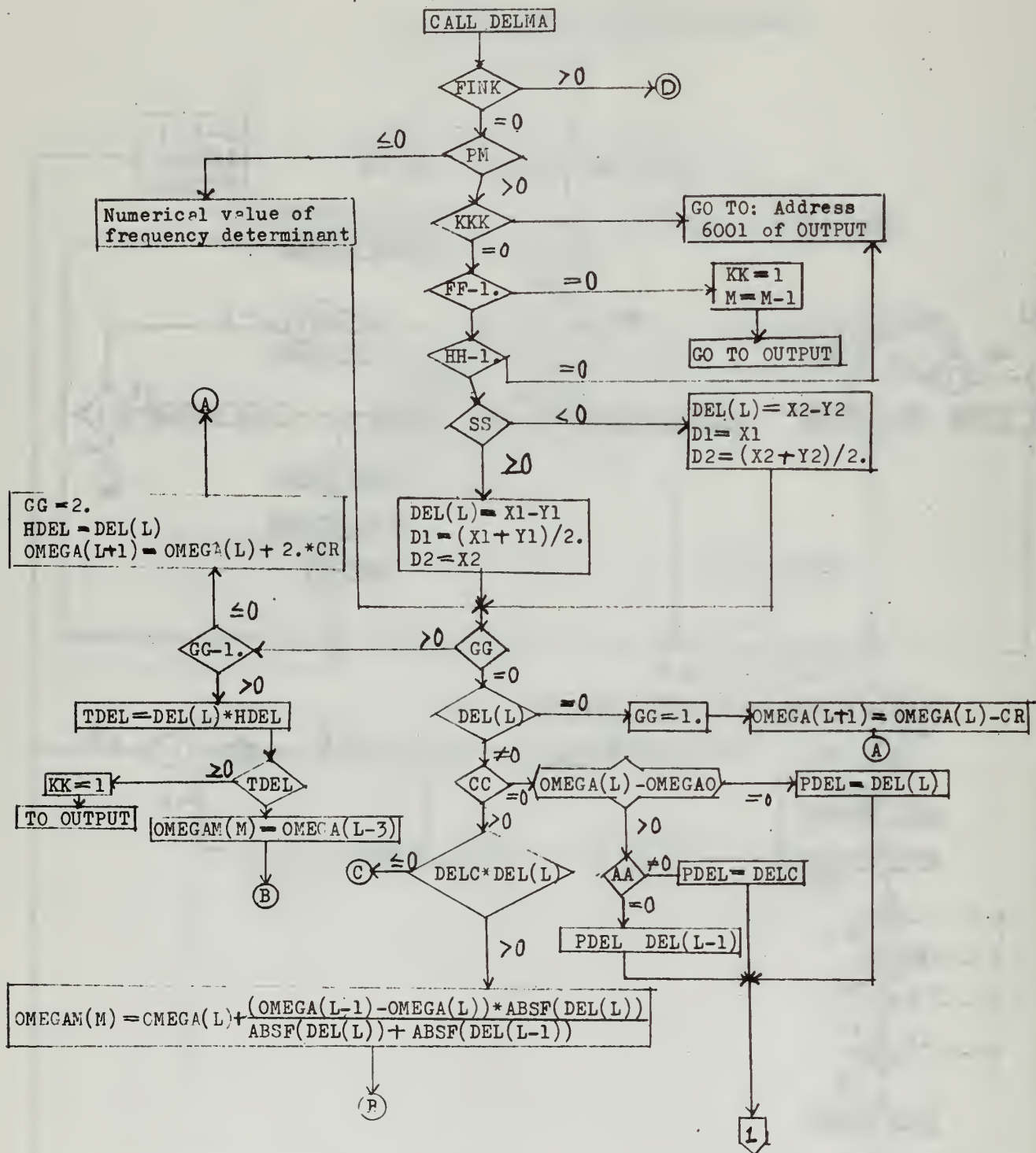
CONTROL IN PLANE FLOW DIAGRAM



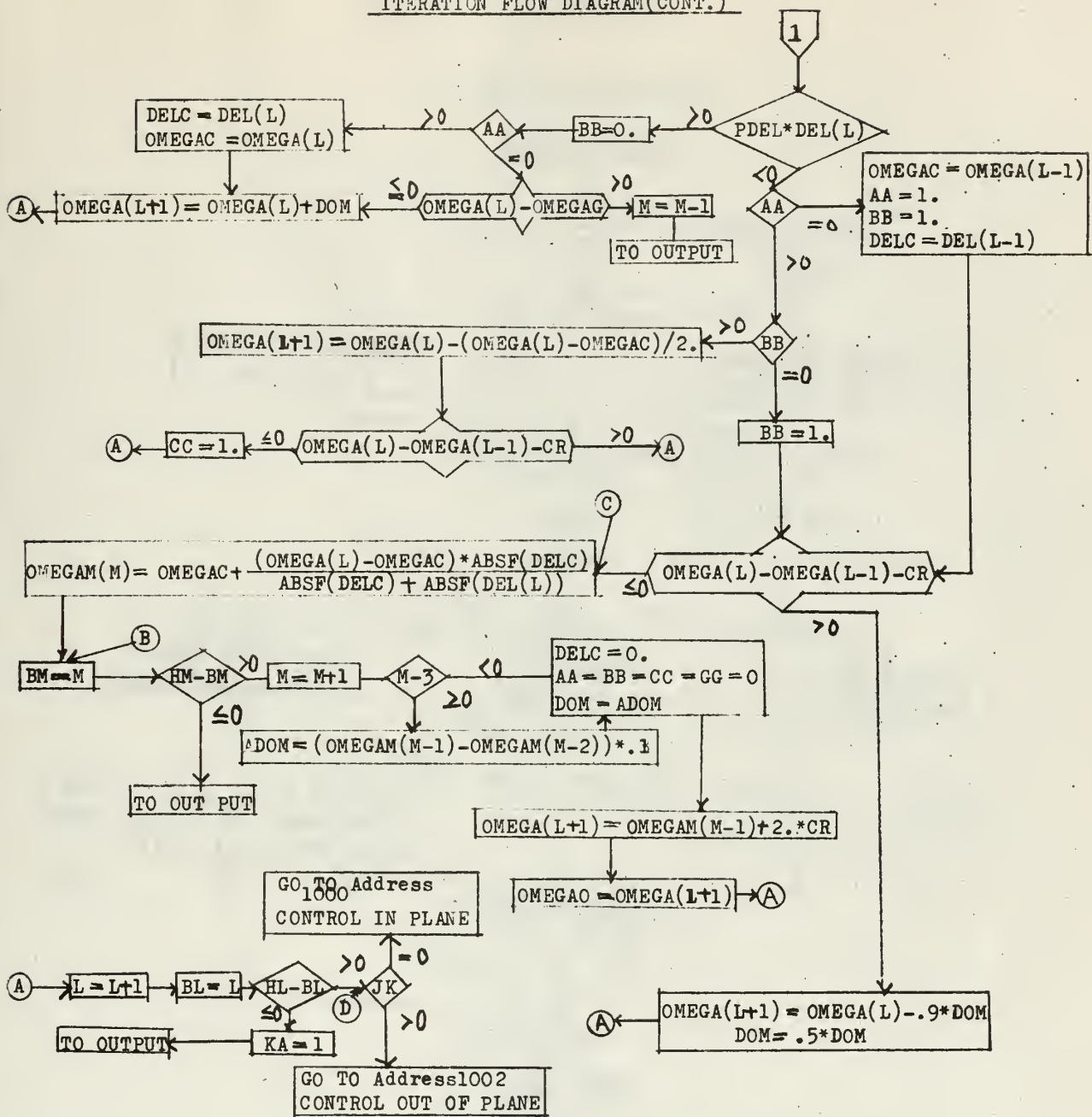
CONTROL IN PLANE DIAGRAM (CONT.)



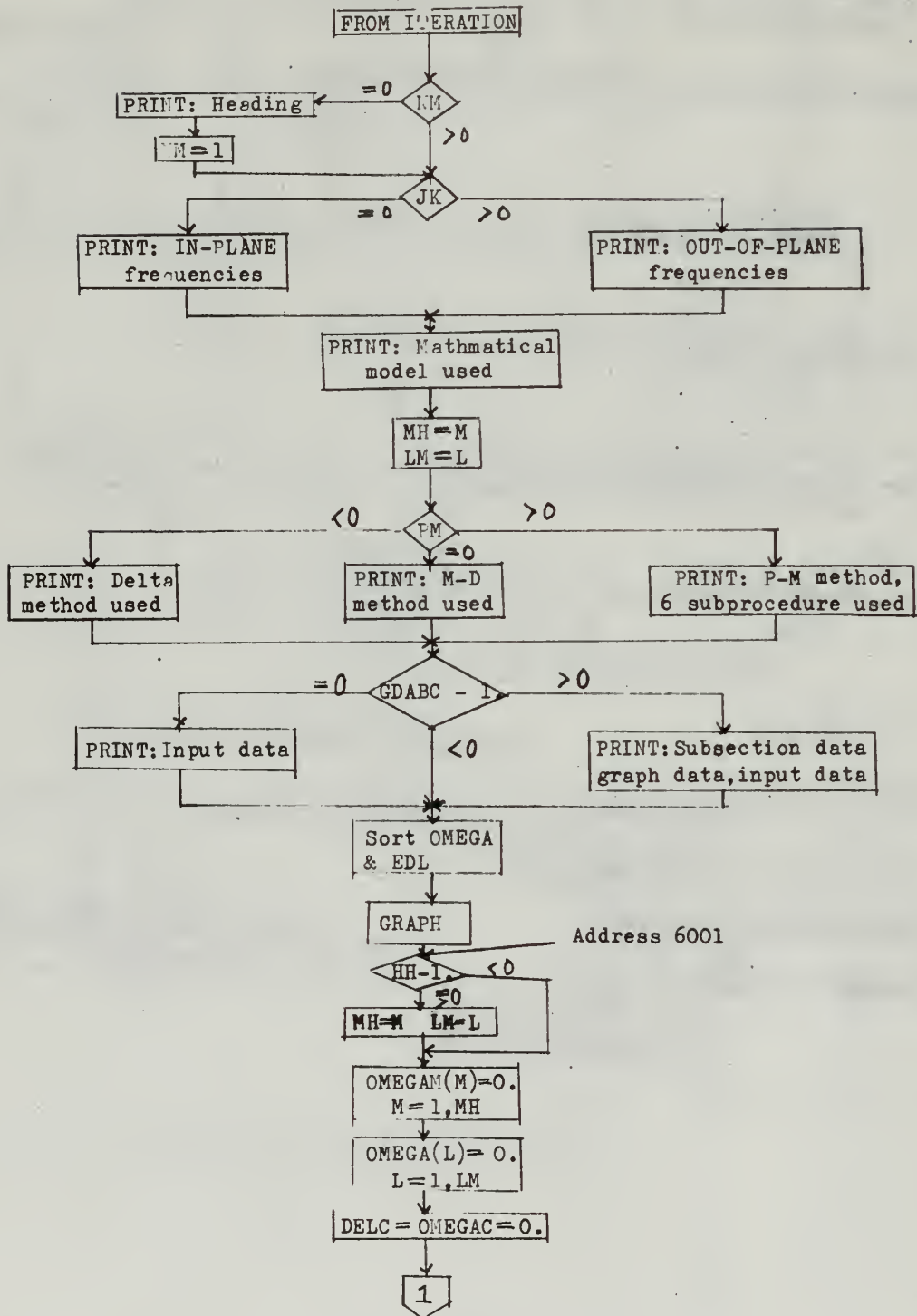
ITERATION FLOW DIAGRAM



ITERATION FLOW DIAGRAM(CONT.)



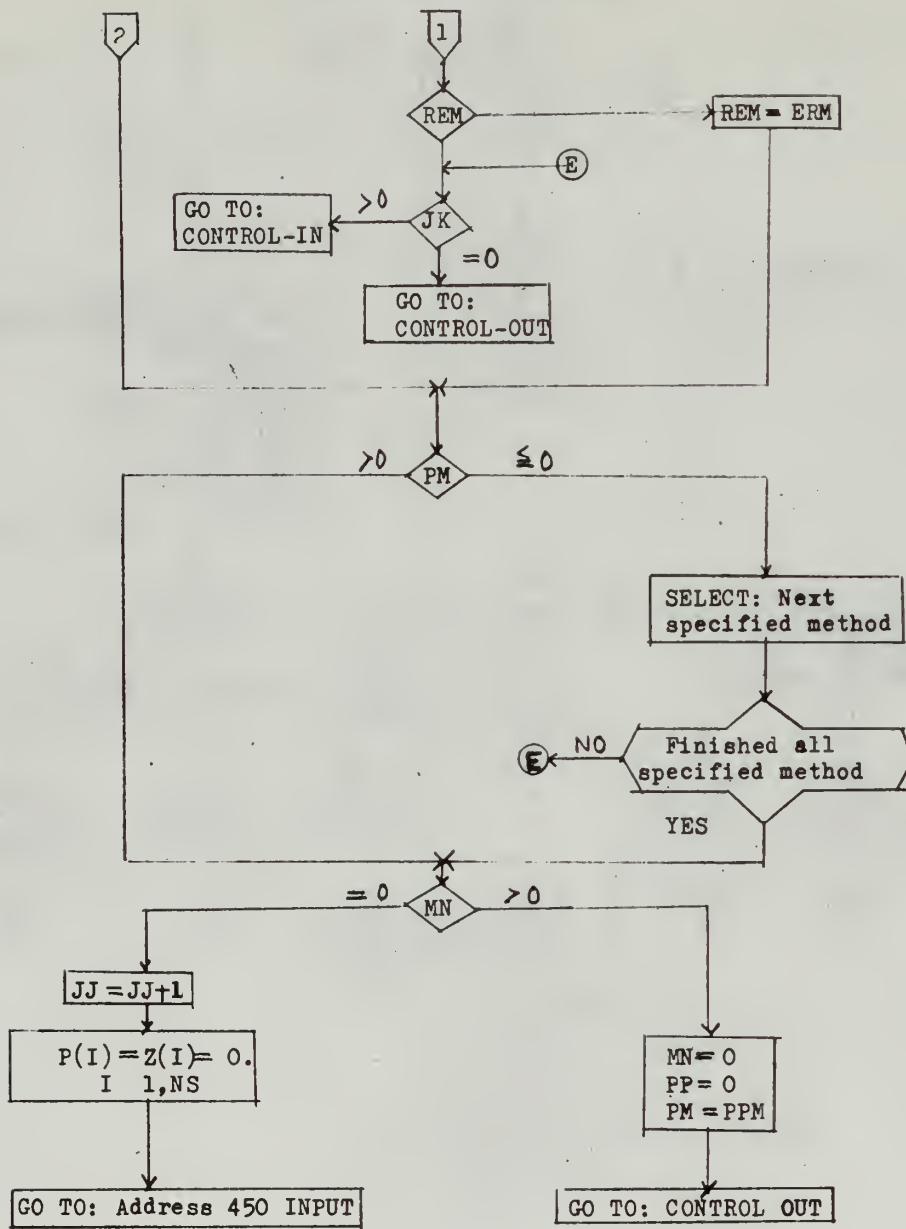
OUTPUT FLOW DIAGRAM



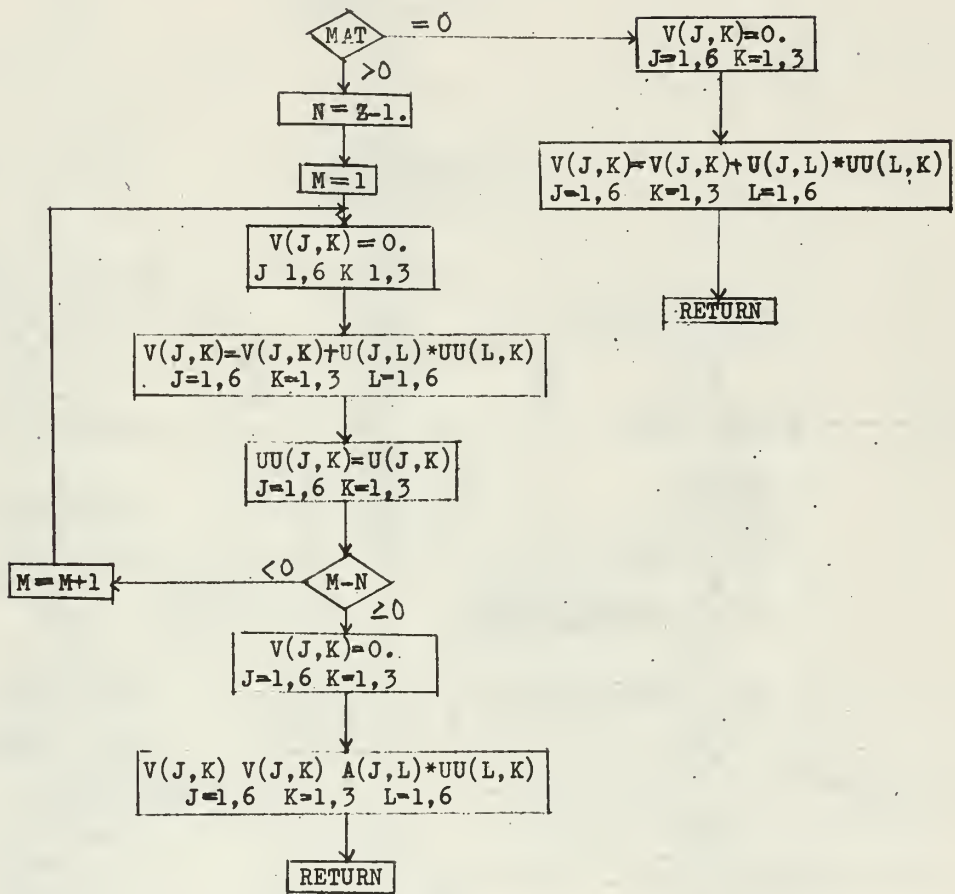
OUTPUT FLOW DIAGRAM (CONT.)



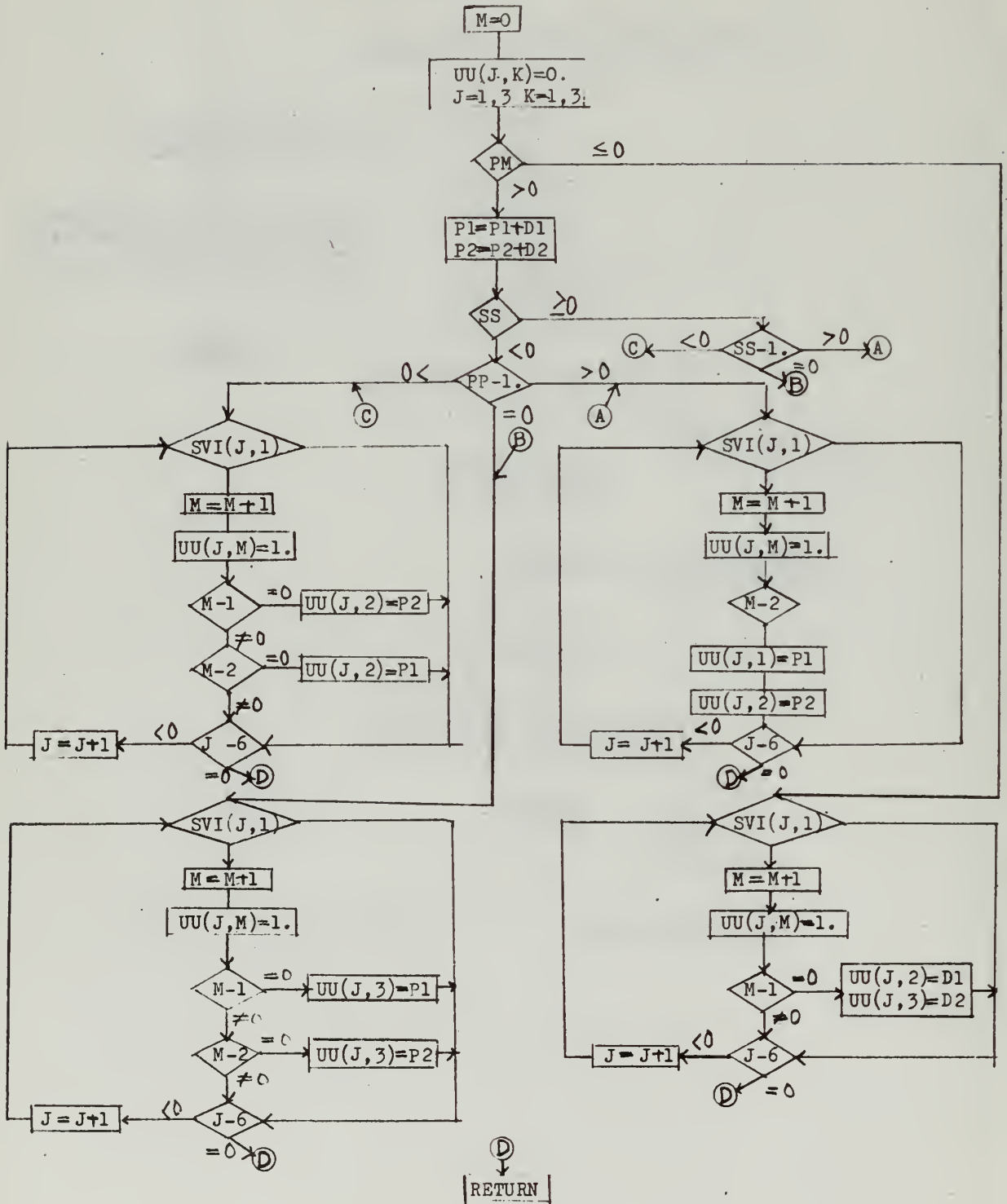
OUTPUT FLOW DIAGRAM(CONT.)



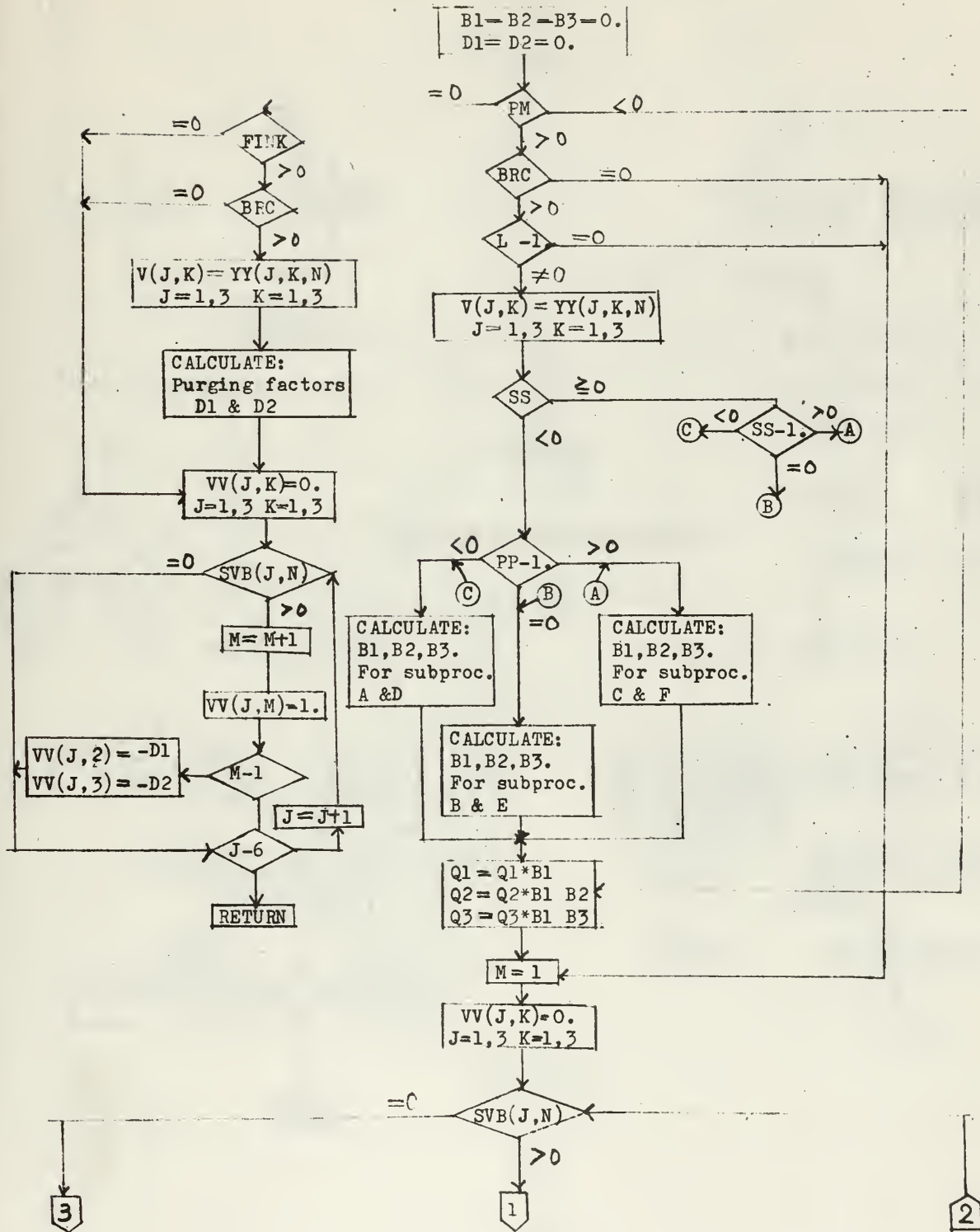
SUBROUTINE FINMAT FLOW DIAGRAM



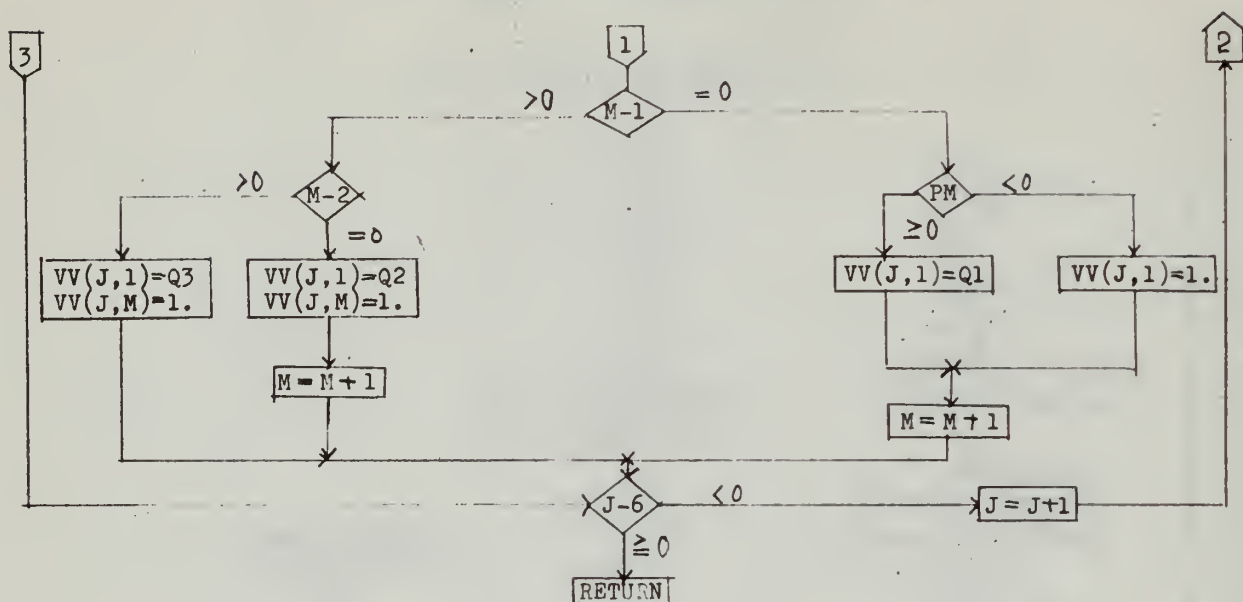
SUBROUTINE STATEMENT FLOW DIAGRAM



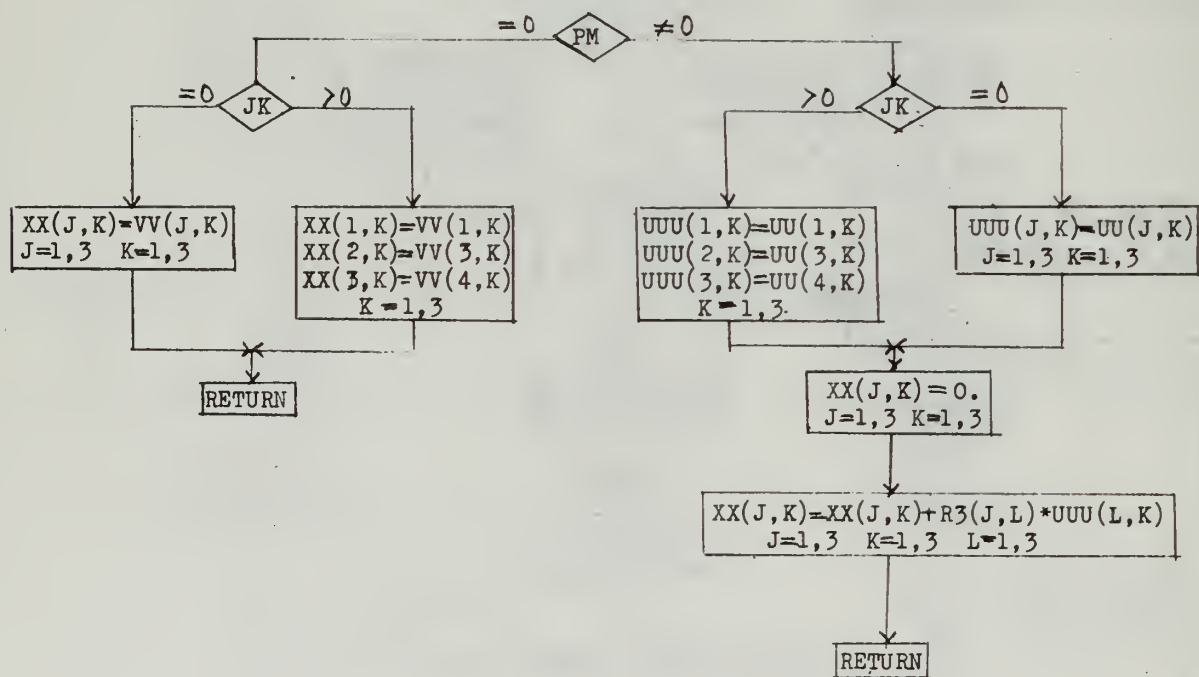
SUBROUTINE STATEB FLOW DIAGRAM



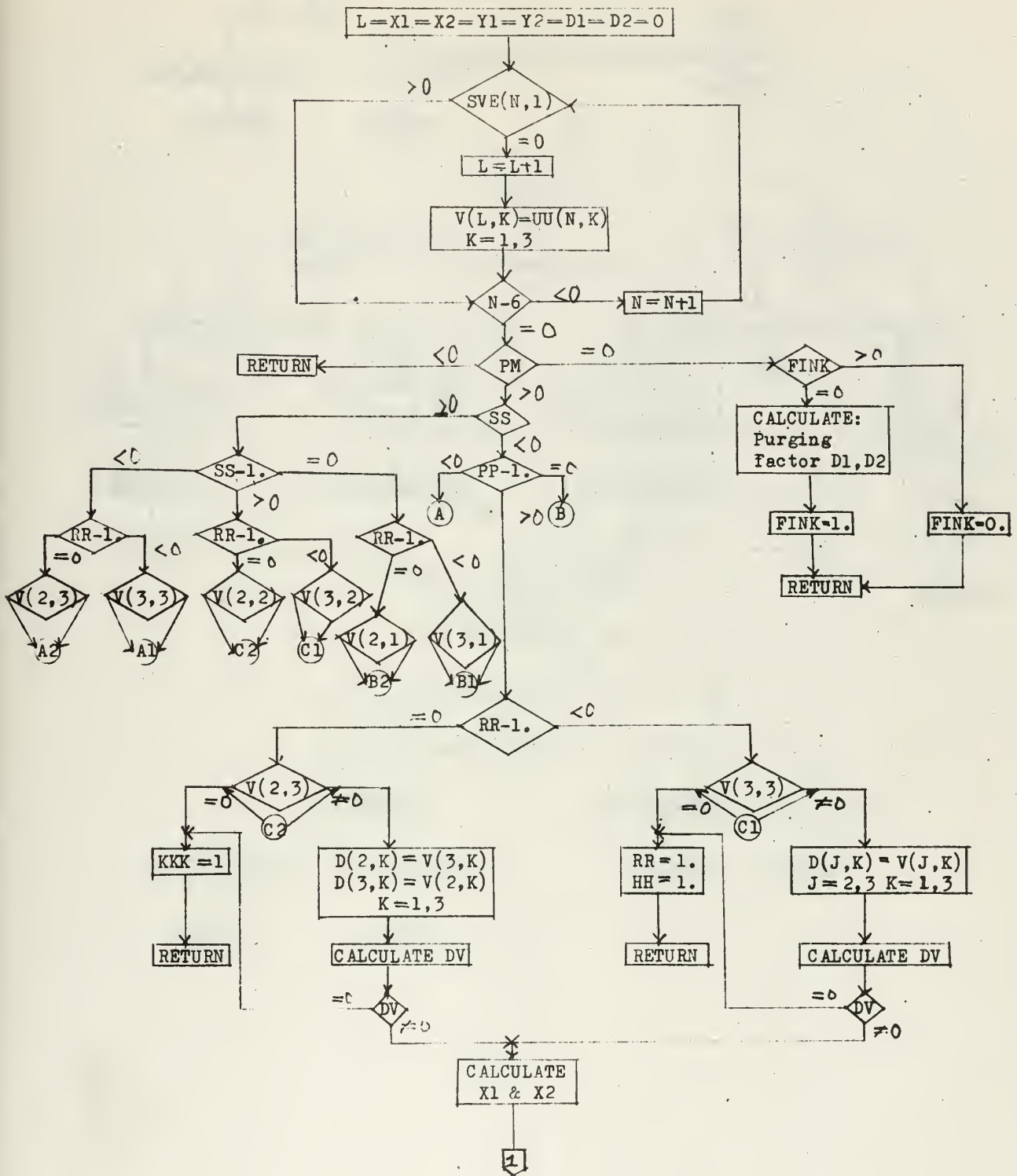
SUBROUTINE STATEB LOW DIAGRAM(CONT.)

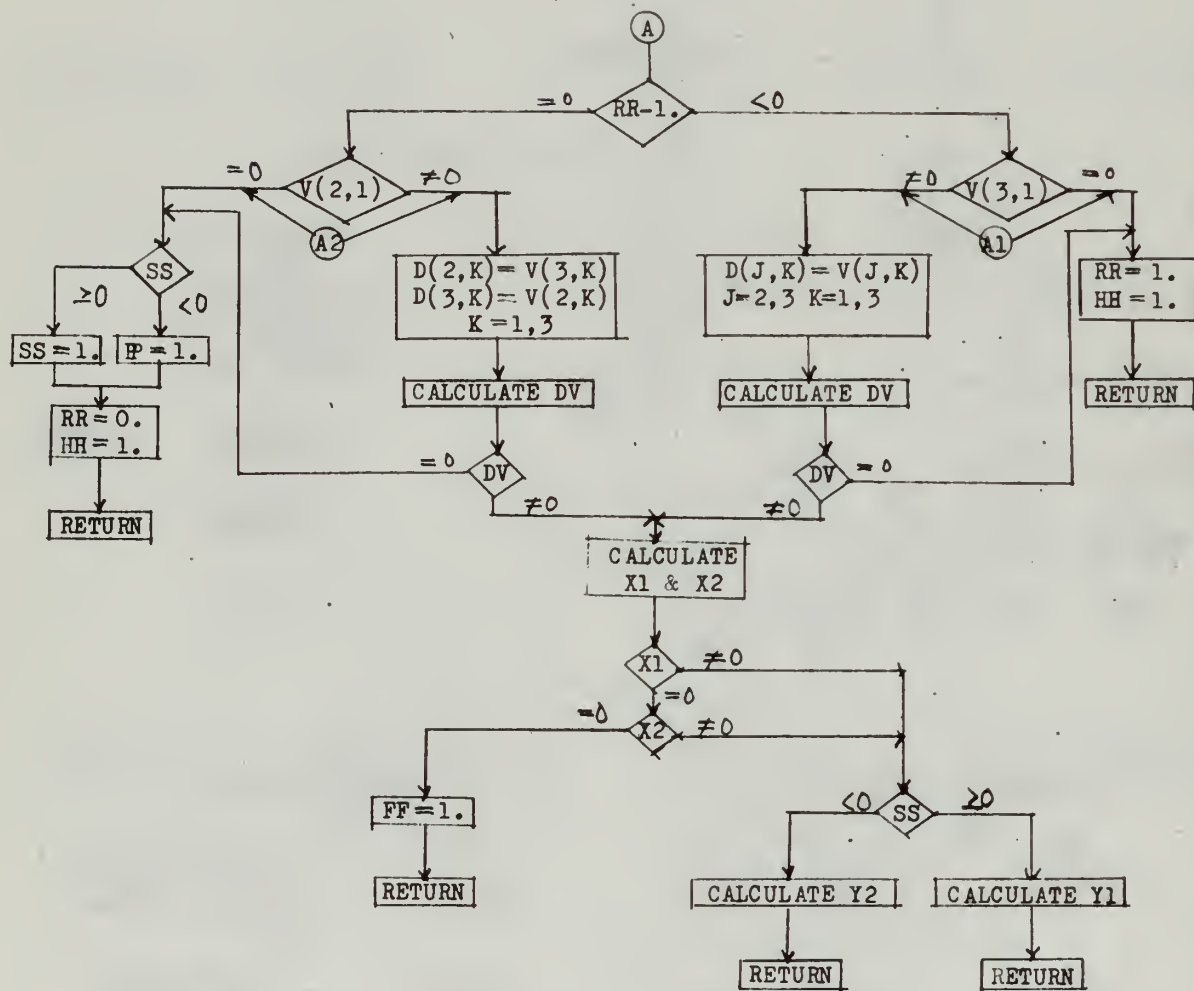
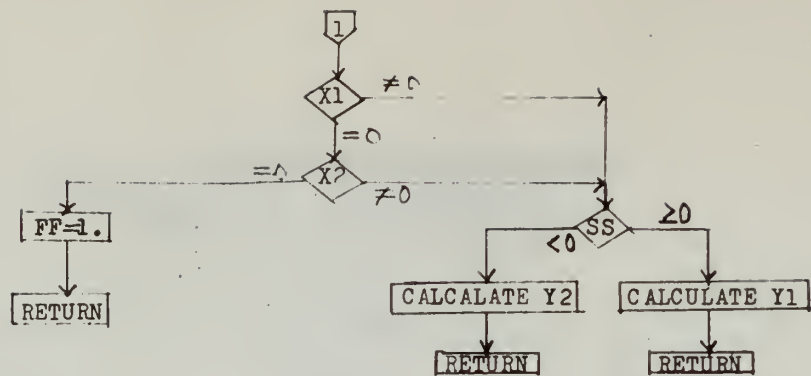


SUBROUTINE BRCOR FLOW DIAGRAM

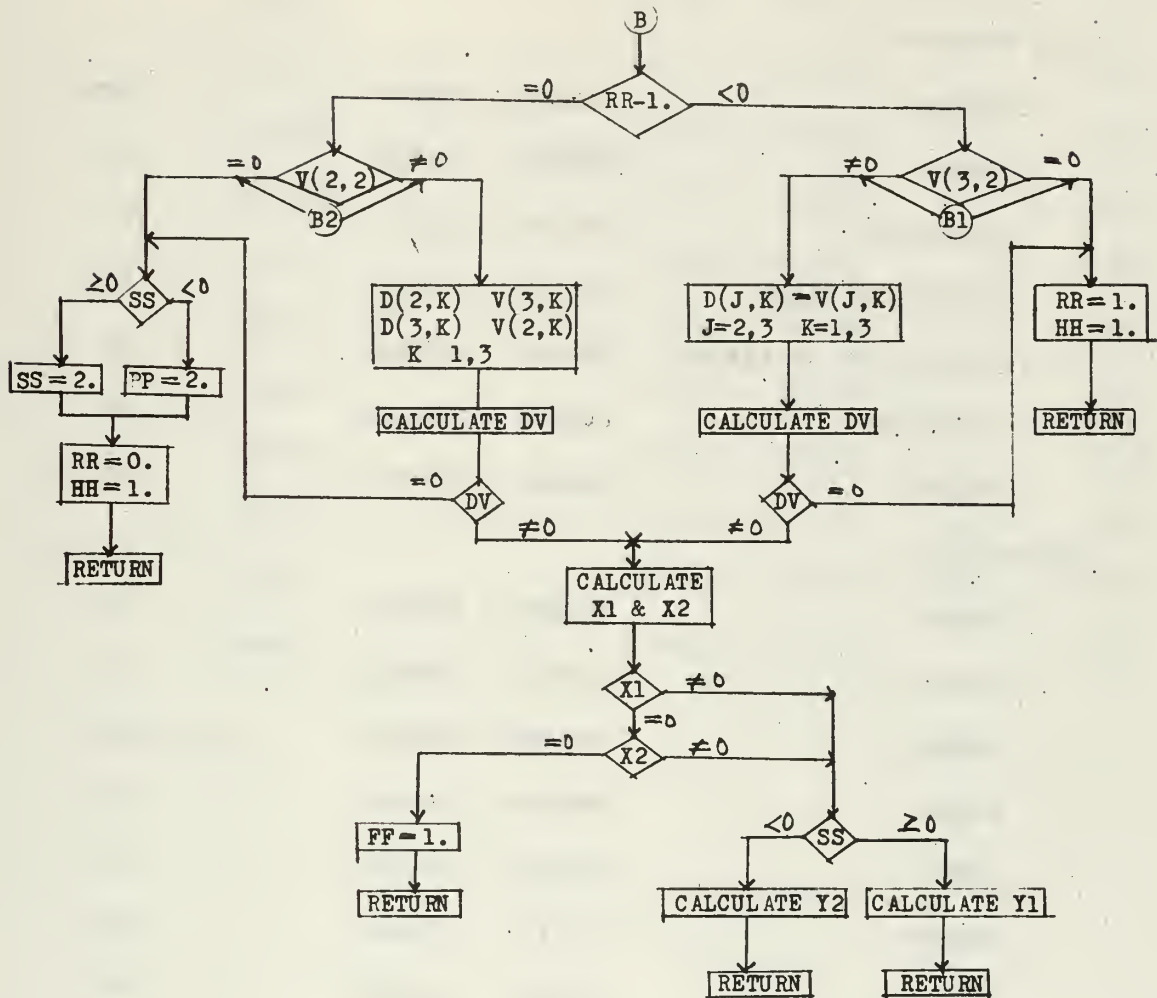


SUBROUTINE DELMA FLOW DIAGRAM





SUBROUTINE DELMA FLOW DIAGRAM(CONT.)



APPENDIX D
PROGRAM LISTING

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D-2 Listing.

	PROGRAM VIPIPE	000000
C		000010
C		000020
C	INPUT	000030
C		000040
		000050
	DIMENSION AJT(50),AJ(50),R(50),AIX(50),AIY(50),PHI(20),SVI(6,1),	000060
	1DT(50),D(50),DI(50),TL(50),Z(50),G(50),E(50),AMU(50),THETA(50),RHO	000070
	2(50),OMEGAM(20),DEL(600),OMEGA(600),A(6,6),B(6,6),U(6,6),CLX(50),C	000080
	3LY(50),CLZ(50),CTX(50),CTY(50),CTZ(50),V(6,3),VV(6,3),UU(6,3),SVE(000090
	46,1),SV(22),SVIP(6,22),SVO(22),SVOP(6,22),NNBR(50),BAMU(50),THETRI	000100
	520),P(20),YY(3,3,23),R3(3,3),XX(3,3),ITITLE(12),EDL(600)	000110
	TYPE DOUBLE VV,UU,V,X2,X1,Y2,Y1,P1,P2,D1,D2,DEL	000120
	NM=0	000130
	JJ=1	000140
	450 READ 1500,NS,NBR,IOP,HM,AMYK,SD,RI,OMEGAG,OMEGAI,GDABC,BRC,GRAPH,	000150
	1REM,PMM,HL	000160
	1500 FORMAT(3I2,4F3.0,2F8.0,4F2.0,2F4.0)	000170
	IF (NS) 600,600,601	000180
	601 NMBR=NBR+2	000190
	IF(IOP-1)1501,1501,1506	000200
	1501 READ 1502,(SV(I),I=1,NMBR)	000210
	1502 FORMAT(22F3.0)	000220
	DO 1505 J=1,NMBR	000230
	IF(SV(J))1503,1503,1505	000240
	1503 READ 1504,(SVIP(I,J),I=1,6)	000250
	1504 FORMAT(6F2.0)	000260
	1505 CONTINUE	000270
	IF(IOP) 1506,1506,1523	000280
	1506 READ 1507,(SVO(I),I=1,NMBR)	000290
	1507 FORMAT(22F3.0)	000300
	DO 1510 J=1,NMBR	000310
	IF(SVO(J))1508,1508,1510	000320
	1508 READ 1509,(SVOP(I,J),I=1,6)	000330
	1509 FORMAT(6F2.0)	000340
	1510 CONTINUE	000350
	1523 IF(NBR) 1513,1513,1511	000360
	1511 READ 1512,(PHI(I),I=1,NBR)	000370
	1512 FORMAT (10F7.3)	000380
	1513OREAD 1514,(D(I),DI(I),RHO(I),TL(I),THETA(I),CLX(I),CLY(I),CLZ(I),C	000390
	1TX(I),CTY(I),CTZ(I),NNBR(I),I=1,NS)	000400
	1514 FORMAT(3F5.2,2F6.2,6F7.2,I2)	000410
	READ 1515 , AAMU,AE,AG,DD	000420
	1515 FORMAT (F7.3,2E10.2,F2.0)	000430
	IF(DD)1516,1516,1518	000440
	1516 DO 1517 I=1,NS	000450
	AMU(I)=AAMU	000460
	E(I)=AE	000470
	1517 G(I)=AG	000480
	GO TO 1520	000490
	1518 AMU(1)=AAMU	000500
	E(1)=AE	000510
	G(1)=AG	000520
	READ 1519,(AMU(I),E(I),G(I),I=2,NS)	000530
	1519 FORMAT (F7.3,2E10.2)	000540
	1520 IF(OMEGAI) 1121,1121,1521	000550

1121	OMEGA I=.01	000560
1521	OMEGA(1)=OMEGA I	000570
	AM YK=AM YK-1.0	000580
	OMEGA O=OMEGA I	000590
	FFAC=.5	000600
	L=1	000610
	M=1	000620
	MN=0	000630
	AA=0.0	000640
	BB=0.0	000650
	CC=0.0	000660
	FF=0.0	000670
	GG=0.0	000680
	RR=0.0	000690
	KK=0	000700
	PP=0.	000710
	KA=0	000720
	KKK=0	000730
	AAMU=0.0	000740
	EE=0.0	000750
	AL=0.0	000760
	AD=0.0	000770
	NNN=NBR+1	000780
	IF(PMM*0.1-1.) 6091,6092,6093	000790
6091	PM=-1.	000800
	GO TO 7000	000810
6092	PM=0.	000820
	GO TO 7000	000830
6093	IF(PMM*0.01-1.) 6094,6095,6096	000840
6094	PM=-1.	000850
	GO TO 7000	000860
6095	PM=1.	000870
	GO TO 7000	000880
6096	IF(PMM*0.01-1.1) 6097,6098,6099	000890
6097	PM=-1.	000900
	GO TO 7000	000910
6098	PM=0.	000920
	GO TO 7000	000930
6099	PM=-1.	000940
7000	PPM=PM	000950
	IF(PM) 7073,7073,7001	000960
7001	IF(REM-1.) 7071,7072,7071	000970
7071	SS=-1.	000980
	GO TO 7073	000990
7072	SS=0.	001000
7073	DO 1522 I=1,NS	001010
	DT(I)=DI(I)	001020
	BAMU(I)=AMU(I)	001030
	THETR(I)=THETA(I)	001040
1522	DI(I)=D(I)-2.0*DI(I)	001050
C		001060
C		001070
C	INVARIANTS	001080
C		001090
	DO 108 I=1,NS	001100
	IF(CLX(I)+CLY(I)+CLZ(I)+CTX(I)+CTY(I)+CTZ(I)) 108,101,108	001110

101	AMU(I)=AMU(I)*3.141592654*(D(I)**2-DI(I)**2)/(6912.*32.17*12.)	001120
	IF(RHO(I))102,103,102	001130
102	THETA(I)=THETA(I)*3.141592654/180.	001140
	TL(I)=THETA(I)*ABSF(RHO(I))	001150
103	AJT(I)=3.141592654*(D(I)**4-DI(I)**4)/32.	001160
	AJ(I)=AJT(I)/2.	001170
	R(I)=1.0/(AJ(I)*E(I))	001180
	AIX(I)=SQRTF((D(I)**2+DI(I)**2)/8.)	001190
	AIY(I)=AIX(I)/1.414214	001200
	IF(AMYK)104,105,104	001210
104	Z(I)=1.0	001220
	IF(RHO(I))105,106,105	001230
105	HMM=HM	001240
	HMH=0.	001250
	CALL SUBSEC (RHO(I),THETA(I),D(I),TL(I),HM,Z(I),OMEGA1,HMH)	001260
	HM=HMH	001270
106	IF(NNBR(I))108,107,108	001280
107	AL=AL+TL(I)	001290
	AD=AD+D(I)*TL(I)	001300
	EE=EE+E(I)*TL(I)	001310
	AID=AID+DI(I)*TL(I)	001320
	AAMU=AAMU+AMU(I)*TL(I)	001330
108	CONTINUE	001340
	AD=AD/AL	001350
	AID=AID/AL	001360
	AFR=3.141592654/64.*(AD**4-AID**4)	001370
	FR=4.73004019**2*SQRTF(EE*AFR/(AAMU*AL**4))	001380
	ADOM=FR/4.0	001390
	DOM=ADOM	001400
	CR=ADOM*0.001	001410
	BDOM=ADOM	001420
	BR=CR	001430
	ERM=REM	001440
	DO 109 II=1,NBR	001450
109	PHI(II)=PHI(II)*3.141592654/180.0	001460
	IF(IOP-1) 110,999,998	001470
110	MN=1	001480
C		001490
C		001500
C	CONTROL IN PLANE	001510
C		001520
999	P1=1.0	001530
	P2=1.0	001540
	D1=0.	001550
	D2=0.	001560
	Q1=1.0	001570
	Q2=1.0	001580
	Q3=1.0	001590
	B1=0.	001600
	B2=0.	001610
	B3=0.	001620
	FINK=0.	001630
	IF(SV(1))1011,1021,1011	001640
1011	CALL STAVEC (SV(1),1,SV1)	001650
	GO TO 1041	001660
1021	DO 1031 J=1,6	001670

1031	SVI(J,1)=SVIP(J,1)	001680
1041	IF(SV(NMBR))1051,1061,1051	001690
1051	CALL STAVEC(SV(NMBR),1,SVE)	001700
	GO TO 1081	001710
1061	DO 1071 J=1,6	001720
1071	SVE(J,1)=SVIP(J,NMBR)	001730
1081	JK=0	001740
	IF(NBR)1000,1000,1085	001750
1085	DO 1087 N=2,NNN	001760
	IF(SV(N))1087,1087,1086	001770
1086	CALL STAVEC(SV(N),N,SVIP)	001780
1087	CONTINUE	001790
1000	N=2	001800
	II=1	001810
	HH=0.	001820
	DO 1001 I=1,NS	001830
	MAT=0	001840
	BC=0.	001850
	P(I)=CLX(I)+CLY(I)+CTZ(I)	001860
	IF(P(I))37,38,37	001870
37	CALL HANGER(CLX(I),CLY(I),CTZ(I),U)	001880
	GO TO 62	001890
38	IF(AMYK)39,41,39	001900
39	IF(RHO(I))40,42,40	001910
40	IF(Z(I))43,45,43	001920
41	IF(RHO(I))40,48,40	001930
42	IF(Z(I))46,47,46	001940
43	CALL CFIELD (R(I),Z(I),G(I),SD,RHO(I),THETA(I),D(I),DI(I),E(I),A)	001950
44	CALL POINT (AMU(I),AIY(I),OMEGA(L),TL(I),Z(I),RI,B)	001960
	CALL MATMUL (A,B,U)	001970
	MAT=1	001980
	GO TO 62	001990
45	CALL STIFCO (THETA(I),RHO(I),U)	002000
	GO TO 62	002010
460	CALL DISTM (AMU(I),AIY(I),OMEGA(L),TL(I),R(I),D(I),DI(I),G(I),SD,R	002020
	II,E(I),FFAC,U)	002030
	GO TO 62	002040
47	CALL RIGID (AMU(I),TL(I),OMEGA(L),AIY(I),RI,U)	002050
	GO TO 62	002060
48	IF(Z(I))49,47,49	002070
49	CALL SFIELD (DI(I),TL(I),R(I),Z(I),G(I),SD,D(I),E(I),FFAC,A)	002080
	GO TO 44	002090
62	IF(NNBR(I)) 81,80,81	002100
80	IF(I-1)63,79,63	002110
79	CALL STATEM(SVI,UU,D1,D2,P1,P2,PP,PM,SS)	002120
	GO TO 63	002130
63	CALL FINMAT(U,UU,V,MAT,A,Z(I))	002140
	DO 64 J=1,6	002150
	DO 64 K=1,3	002160
64	UU(J,K)=V(J,K)	002170
	IF(PM) 88,1113,84	002180
1113	IF(FINK) 88,84,88	002190
84	IF(BC) 88,88,1111	002200
1111	IF(BRC) 88,88,89	002210
89	CALL BRCOR(R3,UU,XX,JK,VV,PM)	002220
	MMM=N-1	002230

DO 90 J=1,3	002240
DO 90 K=1,3	002250
90 YY(J,K,MMM)=XX(J,K)	002260
88 GO TO 1001	002270
81 IF(NNBR(I)-1) 83,83,65	002280
83 CALL STATEB(SVIP,VV,N,YY,L,BRC,D1,D2,PP,PM,SS,Q1,Q2,Q3,FINK)	002290
65 CALL FINBRA(U,VV,V,MAT,A,Z(I))	002300
DO 66 J=1,6	002310
DO 66 K=1,3	002320
66 VV(J,K)=V(J,K)	002330
IF(NNBR(I))68,67,67	002340
67 IF(NNBR(I)-3)1001,68,68	002350
68 CALL BRANCH(R3,VV,PHI(II),U)	002360
N=N+1	002370
II=II+1	002380
MAT=0	002390
BC=1.	002400
GO TO 63	002410
1001 CONTINUE	002420
GO TO 2000	002430
C	002440
C	002450
C CONTROL OUT OF PLANE	002460
C	002470
998 P1=1.0	002480
P2=1.0	002490
D1=0.	002500
D2=0.	002510
Q1=1.0	002520
Q2=1.0	002530
Q3=1.0	002540
B1=0.	002550
B2=0.	002560
B3=0.	002570
FINK=0.	002580
IF(SVO(1))1013,1023,1013	002590
1013 CALL STAVEO(SVO(1),1,SVI)	002600
GO TO 1043	002610
1023 DO 1033 J=1,6	002620
1033 SVI(J,1)=SVOP(J,1)	002630
1043 IF(SVO(NNBR))1053,1063,1053	002640
1053 CALL STAVEO(SVO(NNBR),1,SVE)	002650
GO TO 1083	002660
1063 DO 1073 J=1,6	002670
1073 SVE(J,1)=SVOP(J,NNBR)	002680
1083 JK=1	002690
IF(NBR)1002,1002,1093	002700
1093 DO 1095 N=2,NNN	002710
IF(SVO(N))1095,1095,1094	002720
1094 CALL STAVEO(SVO(N),N,SVOP)	002730
1095 CONTINUE	002740
1002 N=2	002750
II=1	002760
HH=0.	002770
DO 1003 I=1,NS	002780
BC=0.	002790

MAT=0	002800
P(I)=CTX(I)+CLZ(I)+CTY(I)	002810
IF(P(I))1,2,1	002820
1 CALL HANGE0(CTX(I),CLZ(I),CTY(I),U)	002830
GO TO 14	002840
2 IF(AMYK)3,11,3	002850
3 IF(RHO(I))4,8,4	002860
4 IF(Z(I))6,5,6	002870
5 CALL STIF00 (THETA(I),RHO(I),U)	002880
GO TO 14	002890
60CALL CFIELO(AJT(I),R(I),Z(I),G(I),SD,RHO(I),THETA(I),D(I),DI(I),FF	002900
1AC,A)	002910
7 CALL POINO (AMU(I),AIX(I),AIY(I),OMEGA(L),TL(I),Z(I),RI,B)	002920
CALL MATMUL (A,B,U)	002930
MAT=1	002940
GO TO 14	002950
8 IF(Z(I))10,9,10	002960
9 CALL RIGIO (AMU(I),TL(I),AIX(I),OMEGA(L),AIY(I),AJT(I),RI,U)	002970
GO TO 14	002980
100CALL DISTMO(AMU(I),AIX(I),AIY(I),OMEGA(L),TL(I),AJT(I),R(I),D(I),D	002990
1I(I),G(I),SD,RI,FFAC,U)	003000
GO TO 14	003010
11 IF(RHO(I))4,12,4	003020
12 IF(Z(I))13,9,13	003030
13 CALL SFIELO (DI(I),TL(I),AJT(I),R(I),Z(I),G(I),SD,D(I),FFAC,A)	003040
GO TO 7	003050
14 IF(NNBR(I)) 201,200,201	003060
200 IF(I-1) 15,202,15	003070
202 CALL STATEM(SVI,UU,D1,D2,P1,P2,PP,PM,SS)	003080
GO TO 15	003090
15 CALL FINMAT(U,UU,V,MAT,A,Z(I))	003100
DO 16 J=1,6	003110
DO 16 K=1,3	003120
16 UU(J,K)=V(J,K)	003130
IF(PM) 207,1114,85	003140
1114 IF(FINK) 207,85,207	003150
85 IF(BC) 207,207,1112	003160
1112 IF(BRC) 207,207,208	003170
208 CALL BRCOR(R3,UU,XX,JK,VV,PM)	003180
MMM=N-1	003190
DO 209 J=1,3	003200
DO 209 K=1,3	003210
209 YY(J,K,MMM)=XX(J,K)	003220
207 GO TO 1003	003230
201 IF(NNBR(I)-1) 204,204,17	003240
204 CALL STATEB(SVOP,VV,N,YY,L,BRC,D1,D2,PP,PM,SS,Q1,Q2,Q3,FINK)	003250
17 CALL FINBRA(U,VV,V,MAT,A,Z(I))	003260
DO 18 J=1,6	003270
DO 18 K=1,3	003280
18 VV(J,K)=V(J,K)	003290
IF(NNBR(I))82,19,19	003300
19 IF(NNBR(I)-3)1003,82,82	003310
82 CALL BRANCO(R3,VV,PHI(II),U)	003320
N=N+1	003330
II=II+1	003340
MAT=0	003350

BC=1.	003360
GO TO 15	003370
1003 CONTINUE	003380
C	003390
C ITERATION	003400
C	003410
2000 CALL DELMA(UU,SVE,V,X2,Y2,KKK,X1,FF,PP,HH,RR,SS,Y1,REM,FINK,PM,D1,	003420
1D2)	003430
IF(FINK) 87,87,86	003440
87 IF(PM) 99,99,98	003450
98 IF(KKK) 7030,7030,301	003460
301 GO TO 6001	003470
7030 IF(FF-1.0) 7051,7060,7051	003480
7060 KK=1	003490
M=M-1	003500
GO TO 3000	003510
7051 IF(HH-1.0) 300,6001,300	003520
300 IF(SS) 303,302,302	003530
303 DEL(L)=X2-Y2,	003540
D1=X1	003550
D2=(X2+Y2)/2.0	003560
GO TO 61	003570
302 DEL(L)=X1-Y1	003580
D1=(X1+Y1)/2.0	003590
D2=X2	003600
GO TO 61	003610
990DEL(L)=V(1,1)*V(2,2)*V(3,3)-V(1,3)*V(2,2)*V(3,1)+V(1,2)*V(2,3)*V(3	003620
1,1)-V(1,2)*V(2,1)*V(3,3)+V(1,3)*V(3,2)*V(2,1)-V(1,1)*V(3,2)*V(2,3)	003630
GO TO 61	003640
21 PDEL=DEL(L)	003650
GO TO 25	003660
22 IF(AA)23,23,24	003670
23 PDEL=DEL(L-1)	003680
GO TO 25	003690
24 PDEL=DELC	003700
25 IF(PDEL*DEL(L))26,26,27	003710
26 IF(AA)28,28,57	003720
27 BB=0	003730
IF(AA)35,35,52	003740
28 AA=1.0	003750
BB=1.0	003760
DELC=DEL(L-1)	003770
OMEGAC=OMEGA(L-1)	003780
GO TO 59	003790
290OMEGAM(M)=OMEGAC+(OMEGA(L)-OMEGAC)*ABSF(DELC)/(ABSF(DELC)+	003800
1ABSF(DEL(L)))	003810
GO TO 32	003820
30 OMEGA(L+1)=OMEGA(L)-0.9*DOM	003830
DOM=0.5*DOM	003840
31 L=L+1	003850
BL=L	003860
IF(HL-BL) 7053,7053,86	003870
7053 KA=1	003880
GO TO 3000	003890
86 IF(JK)1000,1000,1002	003900
32 BM=M	003910

IF(HM-BM)3000,3000,33	003920
33 M=M+1	003930
IF(M-3) 340,34,34	003940
34 ADOM=(OMEGAM(M-1)-OMEGAM(M-2))*0.1	003950
340 AA=0.0	003960
BB=0	003970
CC=0	003980
GG=0.0	003990
DOM=ADOM	004000
DELC=0	004010
OMEGA(L+1)=OMEGAM(M-1)+2.*CR	004020
OMEGAO=OMEGA(L+1)	004030
GO TO 31	004040
35 IF(OMEGA(L)-OMEGAG)53,60,60	004050
50 OMEGA(L+1)=OMEGA(L)-(OMEGA(L)-OMEGAC)/2.0	004060
IF(OMEGA(L)-OMEGA(L+1)-CR)51,51,31	004070
51 CC=1.0	004080
GO TO 31	004090
52 DELC=DEL(L)	004100
OMEGAC=OMEGA(L)	004110
53 OMEGA(L+1)=OMEGA(L)+DOM	004120
GO TO 31	004130
54 IF(DELC*DEL(L))29,29,55	004140
550 OMEGAM(M)=OMEGA(L)+(OMEGA(L-1)-OMEGA(L))*ABSF(DEL(L))/(ABSF(DEL(L)	004150
1)+ABSF(DEL(L-1)))	004160
GO TO 32	004170
56 IF(OMEGA(L)-OMEGAO)21,21,22	004180
57 IF(BB)58,58,50	004190
58 BB=1.0	004200
59 IF(OMEGA(L)-OMEGA(L-1)-CR)29,29,30	004210
60 M=M-1	004220
GO TO 3000	004230
61 IF(GG)70,70,72	004240
70 IF(DEL(L))77,71,77	004250
71 GG=1.0	004260
OMEGA(L+1)=OMEGA(L)-CR	004270
GO TO 31	004280
72 IF(GG-1.0)73,73,74	004290
73 GG=2.0	004300
HDEL=DEL(L)	004310
OMEGA(L+1)=OMEGA(L)+2.*CR	004320
GO TO 31	004330
74 TDEL=DEL(L)*HDEL	004340
IF(TDEL)75,76,76	004350
75 OMEGAM(M)=OMEGA(L-2)	004360
GO TO 33	004370
76 KK=1	004380
M=M-1	004390
GO TO 3000	004400
77 IF(CC)56,56,54	004410
C	004420
C OUTPUT	004430
C	004440
3000 IF(NM)400,400,403	004450
400 PRINT 401	004460
401 FORMAT (1H1//)	004470

PRINT 402	004480
402 FORMAT (15H PROGRAM VIPIPE,35X, 15HY. S. KIM NHA-3,38X, 13H MARCH	004490
1 1966//,10X,108H MODAL FREQUENCIES OF A PLANAR PIPING	004500
2 SYSTEM ARE DETERMINED BY AN ITERATIVE PROCEDURE USING THE /29H ME	004510
3THOD OF TRANSFER MATRICES.///119H *****	004520
4*****	004530
5***** //	004540
403 IF(NM)4058,4058,404	004550
404 PRINT 405	004560
405 FORMAT (1H1//)	004570
4058 NM=1	004580
4059 IF(JK)4060,4060,4065	004590
4060 PRINT 4070,JJ	004600
40700FORMAT (50X,8HPROBLEM I2,//35X,47HIN PLANE MODE FREQUENCIES (RADIA	004610
1NS PER SECOND) //45X,4HMODE,15X,10H FREQUENCY /)	004620
GO TO 4079	004630
4065 PRINT 4075,JJ	004640
40750FORMAT(50X,8HPROBLEM I2,//33X,51HOUT OF PLANE MODE FREQUENCIES (RA	004650
1DIANS PER SECOND) //45X,4HMODE,15X,10H FREQUENCY /)	004660
4079 MH=M	004670
PRINT 408,(M,OMEGAM(M),M=1,MH)	004680
408 FORMAT (46X,I2,17X,F14.8 /)	004690
IF(KA) 5000,5000,5010	004700
5010 PRINT 5020	004710
5020 FORMAT(10X,53HPROBLEM TERMINATED DUE TO OMEGA STORAGE LIMITATION.	004720
1 /)	004730
5000 IF(KK)4080,4090,4080	004740
4080 PRINT 4085	004750
40850FORMAT(10X,69HPROBLEM TERMINATED DUE TO SIGNIFICANT FIGURE LIMITAT	004760
1ION OF COMPUTER. /)	004770
4090 IF(AMK)412,409,412	004780
409 PRINT 411	004790
4110FORMAT (16X,53H A LUMPED MASS APPROACH WAS EMPLOYED, A	004800
1ND)	004810
GO TO 414	004820
412 PRINT 413	004830
413 FORMAT (24X,45HA DISTRIBUTED MASS APPROACH WAS EMPLOYED AND)	004840
414 IF(SD)415,415,420	004850
415 IF(RI)416,416,418	004860
416 PRINT 417	004870
4170FORMAT (2X,67HTHE EFFECTS OF SHEAR DEFLECTION AND ROTARY INERTIA W	004880
ERE CONSIDERED. /)	004890
GO TO 4610	004900
418 PRINT 419	004910
4190FORMAT (3X,63HTHE EFFECTS OF SHEAR DEFLECTION WERE CONSIDERED WHIL	004920
LE THOSE OF /3X,35HROTATIONAL INERTIA WERE NEGLECTED. /)	004930
GO TO 4610	004940
420 IF(RI)421,421,423	004950
421 PRINT 422	004960
4220FORMAT (3X,72HTHE EFFECTS OF ROTATIONAL INERTIA WERE CONSIDERED WH	004970
ILE THOSE OF SHEAR /3X,27HDEFLECTION WERE NEGLECTED. /)	004980
GO TO 4610	004990
423 PRINT 424	005000
4240FORMAT(3X,66HTHE EFFECTS OF SHEAR DEFLECTION AND ROTARY INERTIA WE	005010
IRE NEGLECTED. /)	005020
4610 IF(PM) 425,425,4713	005030

4713 IF(SS) 4611,4612,4612	005040
4611 IF(PP-1.) 4613,4614,4615	005050
4613 PRINT 4623	005060
4623 FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(A) USED. /)	005070
GO TO 425	005080
4614 PRINT 4624	005090
4624 FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(B) USED. /)	005100
GO TO 425	005110
4615 PRINT 4625	005120
4625 FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(C) USED. /)	005130
GO TO 425	005140
4612 IF(SS-1.) 4616,4617,4618	005150
4616 PRINT 4626	005160
4626 FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(D) USED. /)	005170
GO TO 425	005180
4617 PRINT 4627	005190
4627 FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(E) USED. /)	005200
GO TO 425	005210
4618 PRINT 4628	005220
4628 FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(F) USED. /)	005230
425 IF(PM) 9010,9020,9000	005240
9010 PRINT 9012	005250
9012 FORMAT(24X,20HDELTA METHOD USED. /)	005260
GO TO 9000	005270
9020 PRINT 9022	005280
9022 FORMAT(24X,29HMODIFIED DELTA METHOD USED. /)	005290
9000 PRINT 440	005300
4400FORMAT(118H *****	005310
1*****)	005320
IF(GDABC-1.)4511,4520,4401	005330
4401 PRINT 4402	005340
44020FORMAT(35H SECTION LENGTH AND SUBSECTION DATA/23X,14HSECTION NUMBE	005350
1R,8X,25HLENGTH OF SECTION(INCHES),5X,21HNUMBER OF SECTIONS)	005360
PRINT 443,(I,TL(I),Z(I),I=1,NS)	005370
443 FORMAT (29X,I3,23X,F7.2,21X,F4.0)	005380
LM=L	005390
IF(PM) 7054,7054,7055	005400
7055 PRINT 441	005410
4410FORMAT (11H GRAPH DATA /22X,16HITERATION NUMBER,4X,30HFREQUENCY (R	005420
1ADIANS PER SECOND),3X,30H VALUE OF REMAINDER)	005430
GO TO 7057	005440
7054 PRINT 7056	005450
70560FORMAT (11H GRAPH DATA /22X,16HITERATION NUMBER,4X,30HFREQUENCY (R	005460
1ADIANS PER SECOND),3X,30HVALUE OF FREQUENCY DETERMINANT)	005470
7057 PRINT 444,(L,OMEGA(L),DEL(L),L=1,LM)	005480
444 FORMAT (29X,I3,23X,F11.6,18X,E15.8)	005490
4520 PRINT 442	005500
442 FORMAT(1X,10HINPUT DATA /)	005510
PRINT 4521	005520
45210FORMAT(1X,9HCOMPONENT,4X,8HDIAMETER,5X,14HWALL THICKNESS ,5X,6HLEN	005530
1GTH,5X,9HRADIUS OF,9X,8HINCLUDED,5X,7HDENSITY,5X,7HELASTIC,5X,5HSH	005540
2EAR /57X,9HCURVATURE,5X,12HANGLE OF ARC,16X,7HMODULUS,5X,7HMODULUS	005550
3 /)	005560
DO 4525 I=1,NS	005570
IF(P(I))4524,4524,4523	005580
4524 PRINT 4522,I,D(I),DT(I),TL(I),RHO(I),THETR(I),BAMU(I),E(I),G(I)	005590

45220	FORMAT (3X,13,8X,F7.3,10X,F6.3,7X,F8.3,4X,F8.3,9X,F6.2,8X,F7.2,4X,	005600
	1E8.2,4X,E8.2)	005610
	GO TO 4525	005620
4523	SH=1.	005630
4525	CONTINUE	005640
	IF(SH)4530,4530,4526	005650
4526	PRINT 4532	005660
4532	FORMAT (//1X,7HHANGERS /)	005670
	PRINT 4527	005680
45270	FORMAT (2X,9HCOMPONENT,14X,3HCLX,14X,3HCLY,14X,3HCLZ,14X,3HCTX,14X	005690
	1,3HCTY,14X,3HCTZ /)	005700
4528	DO 4530 I=1,NS	005710
	IF(P(I))4530,4530,4529	005720
4529	PRINT 4531,I,CLX(I),CLY(I),CLZ(I),CTX(I),CTY(I),CTZ(I)	005730
4531	FORMAT(3X,13,15X,F10.2,7X,F10.2,7X,F10.2,7X,F10.2,7X,F10.2,7X,F10.	005740
	12)	005750
4530	CONTINUE	005760
	PRINT 448	005770
448	FORMAT (//20H BOUNDARY CONDITIONS)	005780
	PRINT 449,(SVI(I,1),I=1,6)	005790
449	FORMAT (5H SVI=6F3.0)	005800
	PRINT 451,(SVE(I,1),I=1,6)	005810
451	FORMAT (5H SVE=6F3.0)	005820
	IF(NBR)4511,4511,4502	005830
4502	IF(JK)4505,4505,4515	005840
4505	PRINT 4510,((SVIP(I,J),I=1,6),J=2,NNN)	005850
4510	FORMAT (6F3.0)	005860
	GO TO 4511	005870
4515	PRINT 4517,((SVOP(I,J),I=1,6),J=2,NNN)	005880
4517	FORMAT (6F3.0)	005890
4511	CONTINUE	005900
C	SORT	005910
	IF(GRAPH-1.) 6001,6000,6001	005920
6000	LM1=LM-1	005930
	DO 6100 L=1,LM	005940
	EDL(L)=DEL(L)	005950
6100	CONTINUE	005960
	DO 6600 L=1,LM1	005970
	LP1=L+1	005980
	DO 6600 M=LP1,LM	005990
	IF(OMEGA(L)-OMEGA(M)) 6600,6600,6003	006000
6003	TEMP=OMEGA(L)	006010
	OMEGA(L)=OMEGA(M)	006020
	OMEGA(M)=TEMP	006030
	TEMP=EDL(L)	006040
	EDL(L)=EDL(M)	006050
	EDL(M)=TEMP	006060
6600	CONTINUE	006070
C	GRAPH	006080
	NUMPTS=60	006090
	K=0	006100
	DO 6006 J=1,12	006110
6006	ITITLE(J)=8H	006120
	ITITLE(3)=8H KIM Y S	006130
	ITITLE(8)=8HFREQUENC	006140
	ITITLE(9)=8HY VS R	006150

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        ITITLE(10)=8HEMAINDER
        LT1=LM/NUMPTS
        IF(LT1) 6024,6025,6024
6025  NUMPTS=LM
        LABEL=4H
        L=1
        CALL DRAW(NUMPTS,OMEGA,EDL,0,0,LABEL,ITITLE,0,0,0,0,0,0,9,
        110,K,LAST)
        GO TO 6001
6024  DO 6020 J=1,17
        LT2=0
        LT2=LT2+J*NUMPTS
        IF(LT2-LM) 6020,6021,6022
6020  CONTINUE
6021  LT=LT1
        GO TO 6023
6022  LT=LT1+1
        NUMEND=LM-LT1*NUMPTS
6023  L=1
        DO 7021 I=1,LT
        IF(I-1) 6008,6041,6008
6008  IF(I-LT) 8011,6011,6011
8011  IF(I-3) 6042,6043,8012
8012  IF(I-5) 6044,6045,8013
8013  IF(I-7) 6046,6047,8014
8014  IF(I-9) 6048,6049,8015
8015  IF(I-LT) 6050,6011,6001
6041  LABEL=4H ONE
        GO TO 6009
6042  LABEL=4H TWO
        GO TO 6010
6043  LABEL=4H THRE
        GO TO 6010
6044  LABEL=4H FOUR
        GO TO 6010
6045  LABEL=4H FIVE
        GO TO 6010
6046  LABEL=4H SIX
        GO TO 6010
6047  LABEL=4H SEVN
        GO TO 6010
6048  LABEL=4H EIT
        GO TO 6010
6049  LABEL=4H NINE
        GO TO 6010
6050  LABEL=4H TEN
        GO TO 6010
6009  DO 6031 N=1,NUMPTS
        EDL(N)=EDL(L+N-1)
6031  OMEGA(N)=OMEGA(L+N-1)
        PRINT 6027,NUMPTS,EDL(1)
6027  FORMAT(1H1,////////5X,7HNUMPTS=,I3,5X,7HEDL(1)=,E12.9)
        CALL DRAW(NUMPTS,OMEGA,EDL,0,0,LABEL,ITITLE,0,0,0,0,0,0,9,
        110,K,LAST)
        GO TO 7021
6010  L=L+NUMPTS

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006160
006170
006180
006190
006200
006210
006220
006230
006240
006250
006260
006270
006280
006290
006300
006310
006320
006330
006340
006350
006360
006370
006380
006390
006400
006410
006420
006430
006440
006450
006460
006470
006480
006490
006500
006510
006520
006530
006540
006550
006560
006570
006580
006590
006600
006610
006620
006630
006640
006650
006660
006670
006680
006690
006700
006710

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OMEGAX=OMEGA(L)	006720
DO 6030 N=1,NUMPTS	006730
EDL(N)=EDL(N+L-1)	006740
OMEGA(N)=OMEGA(L+N-1)	006750
6030 OMEGA(N)=OMEGA(N)-OMEGAX	006760
PRINT 6032,NUMPTS,EDL(1)	006770
6032 FORMAT(////////5X,7HNUMPTS=, I3 ,5X,7HEDL(1)=,E12.9)	006780
CALL DRAW(NUMPTS,OMEGA,EDL,0,0,LABEL,ITITLE,0,0,0,0,0,0,9,	006790
110,K,LAST)	006800
GO TO 7021	006810
6011 L=L+NUMPTS	006820
OMEGAX=OMEGA(L)	006830
DO 6033 N=1,NUMEND	006840
EDL(N)=EDL(N+L-1)	006850
OMEGA(N)=OMEGA(N+L-1)	006860
6033 OMEGA(N)=OMEGA(N)-OMEGAX	006870
PRINT 6051,NUMEND,EDL(1)	006880
6051 FORMAT(////////5X,7HNUMEND=,I3, 5X,7HEDL(1)=,E12.9)	006890
LABEL=4HLAST	006900
CALL DRAW(NUMEND,OMEGA,EDL,0,0,LABEL,ITITLE,0,0,0,0,0,0,9,	006910
110,K,LAST)	006920
GO TO 6001	006930
7021 CONTINUE	006940
6001 IF(HH-1.0) 8024,8023,8023	006950
8023 MH=M	006960
LM=L	006970
8024 DO 446 M=1,MH	006980
446 OMEGAM(M)=0.0	006990
DO 447 L=1,LM	007000
OMEGA(L)=0.0	007010
EDL(L)=0.	007020
447 DEL(L)=0.0	007030
DELC=0.0	007040
OMEGAC=0.0	007050
IF(HH-1.0) 8001,8022,8001	007060
8001 IF(PM) 8031,8031,8033	007070
8033 IF(REM) 8031,8032,8032	007080
8031 IF(MN) 8022,8022,452	007090
8032 IF(REM-1.) 8041,8042,8043	007100
8041 IF(PP-1) 8043,8043,8072	007110
8072 IF(SS) 8073,8074,8074	007120
8073 SS=0.	007130
GO TO 8022	007140
8074 IF(SS-1.) 8042,8042,8075	007150
8075 SS=-1.	007160
PP=0.	007170
REM=-1.	007180
GO TO 8033	007190
8042 SS=SS-1.	007200
IF(SS) 8051,8052,8053	007210
8051 SS=1.	007220
GO TO 8022	007230
8052 SS=2.	007240
GO TO 8022	007250
8053 SS=-1.	007260
REM=-1.	007270

GO TO 8033	007280
8043 PP=PP-1.	007290
IF(PP) 8061,8062,8063	007300
8061 PP=1.	007310
GO TO 8022	007320
8062 PP=2.	007330
GO TO 8022	007340
8063 PP=0.	007350
REM=-1.	007360
GO TO 8033	007370
452 PP=0.	007380
RR=0.	007390
SS=-1.	007400
8022 L=1	007410
M=1	007420
AA=0.0	007430
BB=0.0	007440
CC=0.0	007450
FF=0.0	007460
GG=0.0	007470
KK=0	007480
KA=0	007490
KKK=0	007500
ADOM=BDOM	007510
CR=BR	007520
OMEGA(1)=OMEGAI	007530
OMEGAO=OMEGAI	007540
DOM=ADOM	007550
IF(HH-1.0) 8021,7040,8021	007560
7040 IF(SS) 8191,8192,8192	007570
8191 IF(PP-1.) 8091,8092,8093	007580
8192 IF(SS-1.) 8291,8292,8293	007590
8091 PRINT 8081	007600
8081 FORMAT(1H1,////18HMETHOD A FAILED. ///)	007610
GO TO 8021	007620
8092 PRINT 8082	007630
8082 FORMAT(1H1,////18HMETHOD B FAILED. ///)	007640
GO TO 8021	007650
8093 PRINT 8084	007660
8084 FORMAT(1H1,////18HMETHOD C FAILED. ///)	007670
GO TO 8021	007680
8291 PRINT 8281	007690
8281 FORMAT(1H1,////18HMETHOD D FAILED. ///)	007700
GO TO 8021	007710
8292 PRINT 8282	007720
8282 FORMAT(1H1,////18HMETHOD E FAILED. ///)	007730
GO TO 8021	007740
8293 PRINT 8283	007750
8283 FORMAT(1H1,////18HMETHOD F FAILED. ///)	007760
8021 IF(PM) 453,453,8035	007770
8035 IF(REM) 8037,8036,8036	007780
8036 IF(JK) 998,999,998	007790
8037 REM=ERM	007800
453 IF(PM) 6081,6082,6080	007810
6081 IF(PMM-1.) 6084,6080,6084	007820
6084 IF(PMM*0.1-10.1) 6086,6085,6086	007830


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6085 PM=1.
      GO TO 8036
6086 PM=0.
      GO TO 8036
6082 IF(PMM*0.1-1.1) 6080,6080,6085
6080 IF(MN) 9001,9001,8034
8034 MN=0
      PM=PPM
      GO TO 998
9001 JJ=JJ+1
      DO 454 I=1,NS
        P(I)=0.
454  Z(I)=0.0
      GO TO 450
600  END

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C
C      SUBROUTINE SUBSEC (RHO,THETA,D,TL,HM,Z,OMEGAI,HMH)
      RHOD=ABSF(RHO)
      IF(HMH) 41,40,41
40  IF(HM-5.0)8,8,7
8   IF(OMEGAI-800.) 10,7,7
7   HM=6.
      GO TO 10
41  HM=HMH
10  RA=TL/D
      IF(RHOD-0.)3,3,4
3   IF(RA-1.)5,5,6
5   Z=0.
      RETURN
6   IF(RA-3.) 15,15,16
15  Z=2.0
      RETURN
16  IF(RA-6.)17,17,18
17  Z=3.0
      RETURN
18  IF(HM-3.)1,1,2
1   Z=6.0
      RETURN
2   Z=2.0*HM
      RETURN
4   RD=RHOD/D
      IF(RD-1.)19,19,23
19  Z=0
      RETURN
23  IF(RD-3.)24,24,25
24  IF(THETA-.44)26,26,27
26  Z=0.
      RETURN
27  IF(THETA-1.57)28,28,29
28  Z=4.0
      RETURN
29  Z=6.0
      RETURN
25  IF(RD-6.)30,30,31
30  IF(THETA-.44)32,32,33

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007840
007850
007860
007870
007880
007890
007900
007910
007920
007930
007940
007950
007960
007970
007980
007990
008000
008010
008020
008030
008040
008050
008060
008070
008080
008090
008100
008110
008120
008130
008140
008150
008160
008170
008180
008190
008200
008210
008220
008230
008240
008250
008260
008270
008280
008290
008300
008310
008320
008330
008340
008350
008360
008370
008380
008390

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32	Z=4.0	008400
	RETURN	008410
33	IF(THETA-1.57)34,34,35	008420
34	Z=6.0	008430
	RETURN	008440
35	Z=8.0	008450
	RETURN	008460
31	IF(THETA-.44)36,36,3	008470
36	Z=4.0	008480
	RETURN	008490
	END	008500
		008510
		008520
	SUBROUTINE DISTM (AMU,AIY,OMEGA,TL,R,D,DI,G,SD,RI,E,FFAC,U)	008530
	DIMENSION U(6,6)	008540
	ARA=(3.141592654/4.0)*(D**2-DI**2)	008550
	AREA=ARA*FFAC	008560
	BE=TL*OMEGA*SQRTF(AMU/(E*ARA))	008570
	P4=R*AMU*TL**4*OMEGA**2	008580
	IF(RI)1,1,2	008590
1	T=R*AMU*(AIY*OMEGA*TL)**2	008600
	GO TO 3	008610
2	T=0.0	008620
3	IF(SD)4,4,5	008630
4	S=(AMU*(OMEGA*TL)**2)/(G*AREA)	008640
	GO TO 6	008650
5	S=0.0	008660
6	SPT=S+T	008670
	VV=SQRTF(P4+(S-T)**2/4.0)	008680
	AL1=SQRTF(VV-.5*SPT)	008690
	AL2=SQRTF(VV+.5*SPT)	008700
	CL=1.0/(AL1**2+AL2**2)	008710
	CHL1=(EXPF(AL1)+1./EXPF(AL1))/2.	008720
	SHL1=(EXPF(AL1)-1./EXPF(AL1))/2.	008730
	CL2=COSF(AL2)	008740
	SL2=SINF(AL2)	008750
	SBE=SINF(BE)	008760
	CBE=COSF(BE)	008770
	COO=CL*(AL2**2*CHL1+AL1**2*CL2)	008780
	CO1=CL*((AL2**2*SHL1)/AL1+(AL1**2*SL2)/AL2)	008790
	CO2=CL*(CHL1-CL2)	008800
	CO3=CL*(SHL1/AL1-SL2/AL2)	008810
	DO 7 J=1,6	008820
	DO 7 K=1,6	008830
7	U(J,K)=0.0	008840
	U(1,1)=CBE	008850
	U(1,6)=TL*SBE/(BE*ARA*E)	008860
	U(2,2)=COO-S*CO2	008870
	U(2,3)=TL*(CO1-SPT*CO3)	008880
	U(2,4)=R*CO2*TL**2	008890
	U(2,5)=(R*TL**3/P4)*((P4+S**2)*CO3-S*CO1)	008900
	U(3,2)=P4*CO3/TL	008910
	U(3,3)=COO-T*CO2	008920
	U(3,4)=TL*R*(CO1-T*CO3)	008930
	U(3,5)=U(2,4)	008940
	RR=1.0/R	008950

U(4,2)=P4*RR*CO2/TL**2	008960
U(4,3)=RR*((P4+T**2)*CO3-T*CO1)/TL	008970
U(4,4)=U(3,3)	008980
U(4,5)=U(2,3)	008990
U(5,2)=P4*RR*(CO1-S*CO3)/TL**3	009000
U(5,3)=U(4,2)	009010
U(5,4)=U(3,2)	009020
U(5,5)=U(2,2)	009030
U(6,1)=-AMU*TL*OMEGA**2*SBE/BE	009040
U(6,6)=CBE	009050
RETURN	009060
END	009070
	009080
	009090
SUBROUTINE DISTMO (AMU,AIX,AIY,OMEGA,TL,AJT,R,D,DI,G,SD,RI,FFAC,U)	009100
DIMENSION U(6,6)	009110
W=1./(G*AJT)	009120
AREA=(3.141592654/4.0)*(D**2-DI**2)*FFAC	009130
BE=+SQRTF(AMU*(TL*AIX*OMEGA)**2*W)	009140
P4=R*AMU*TL**4*OMEGA**2	009150
IF(RI)2,2,3	009160
2 T=R*AMU*(AIY*OMEGA*TL)**2	009170
GO TO 4	009180
3 T=0.0	009190
4 IF(SD)5,5,6	009200
5 S=(AMU*(OMEGA*TL)**2)/(G*AREA)	009210
GO TO 7	009220
6 S=0.0	009230
7 VV=+SQRTF(P4+(S-T)**2/4.0)	009240
SPT=S+T	009250
AL1=+SQRTF(VV-.5*SPT)	009260
AL2=+SQRTF(VV+.5*SPT)	009270
CL=1.0/(AL1**2+AL2**2)	009280
CHL1=(EXPF(AL1)+1./EXPF(AL1))/2.	009290
SHL1=(EXPF(AL1)-1./EXPF(AL1))/2.	009300
CL2=COSF(AL2)	009310
SL2=SINF(AL2)	009320
SBE=SINF(BE)	009330
CBE=COSF(BE)	009340
CO0=CL*(AL2**2*CHL1 + AL1**2*CL2)	009350
CO1=CL*((AL2**2*SHL1)/AL1 + (AL1**2*SL2)/AL2)	009360
CO2=CL*(CHL1-CL2)	009370
CO3=CL*(SHL1/AL1-SL2/AL2)	009380
DO 1 J=1,6	009390
DO 1 K=1,6	009400
1 U(J,K)=0.0	009410
U(1,1)=CBE	009420
U(1,2)=TL*W*SBE/BE	009430
U(2,1)=-AMU*TL*AIX**2*SBE/BE	009440
U(2,2)=CBE	009450
U(3,3)=CO0-S*CO2	009460
U(3,4)=TL*(CO1-SPT*CO3)	009470
U(3,5)=R*CO2*TL**2	009480
U(3,6)=(R*TL**3/P4)*(-S*CO1+(P4+S**2)*CO3)	009490
U(4,3)=P4*CO3/TL	009500
U(4,4)=CO0-T*CO2	009510

U(4,5)= TL*R*(CO1-T*CO3)	009520
U(4,6)=U(3,5)	009530
RR=1.0/R	009540
U(5,3)=P4*RR*CO2/TL**2	009550
U(5,4)=RR*(-T*CO1+(P4+T**2)*CO3)/TL	009560
U(5,5)=U(4,4)	009570
U(5,6)=U(3,4)	009580
U(6,3)=P4*RR*(CO1-S*CO3)/TL**3	009590
U(6,4)=U(5,3)	009600
U(6,5)=U(4,3)	009610
U(6,6)=U(3,3)	009620
RETURN	009630
END	009640
	009650
	009660
SUBROUTINE RIGID (AMU,TL,OMEGA,AIY,RI,U)	009670
DIMENSION U(6,6)	009680
AM=AMU*TL	009690
DO 1 J=1,6	009700
DO 1 K=1,6	009710
1 U(J,K)=0.0	009720
U(1,1)=1.0	009730
U(6,1)=-AM*OMEGA**2	009740
U(6,6)=1.0	009750
U(2,2)=1.0	009760
U(2,3)=TL	009770
U(3,3)=1.0	009780
U(4,2)=AM*TL*OMEGA**2/2.0	009790
IF(RI)2,2,3	009800
2 U(4,3)=AM*OMEGA**2*(TL**2/6.0-AIY**2)	009810
GO TO 4	009820
3 U(4,3)=AM*(OMEGA*TL)**2/6.0	009830
4 U(4,4)=1.0	009840
U(4,5)=TL	009850
U(5,2)=-U(6,1)	009860
U(5,3)=U(4,2)	009870
U(5,5)=1.0	009880
RETURN	009890
END	009900
	009910
	009920
SUBROUTINE RIGIO (AMU,TL,AIX,OMEGA,AIY,AJT,SDRI,U)	009930
DIMENSION U(6,6)	009940
DO 2 J=1,6	009950
DO 2 K=1,6	009960
2 U(J,K)=0.0	009970
AM=AMU*TL	009980
U(1,1)=1.0	009990
U(2,1)=-AMU*TL*(AIX*OMEGA)**2	010000
U(2,2)=1.0	010010
U(3,3)=1.0	010020
U(3,4)=TL	010030
U(4,4)=1.0	010040
U(5,3)=AM*TL*OMEGA**2/2.0	010050
IF(SDRI-0.) 30,30,31	010060
30 U(5,4)=AM*OMEGA**2*(TL**2/6.0-AIY**2)	010070

```

      GO TO 1
31  U(5,4)=AM*OMEGA**2*(TL**2/6.0)
1   U(5,5)=1.0
      U(5,6)=TL
      U(6,3)=AM*OMEGA**2
      U(6,4)=U(5,3)
      U(6,6)=1.0
      RETURN
      END

```

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010080
010090
010100
010110
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C
C

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      SUBROUTINE STIFCO (THETA,RHO,U)
      DIMENSION U(6,6)
      DO 1 J=1,6
      DO 1 K=1,6
1   U(J,K)=0.0
      IF(RHO)2,2,3
2   V=-1.0
      GO TO 4
3   V=1.0
4   CT=COSE(THETA)
      ST=SINF(THETA)
      U(1,1)=CT
      U(1,2)=-V*ST
      U(2,1)=V*ST
      U(2,2)=CT
      U(3,3)=1.0
      U(4,4)=1.0
      U(5,5)=CT
      U(5,6)=-V*ST
      U(6,5)=V*ST
      U(6,6)=CT
      RETURN
      END

```

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010200
010210
010220
010230
010240
010250
010260
010270
010280
010290
010300
010310
010320
010330
010340
010350
010360
010370
010380
010390
010400
010410
010420
010430

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C
C

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      SUBROUTINE STIFOO (THETA,RHO,U)
      DIMENSION U(6,6)
      IF(RHO)1,1,2
1   V=-1.0
      GO TO 3
2   V=1.0
3   CT=COSE(THETA)
      ST=SINF(THETA)
      DO 4 J=1,6
      DO 4 K=1,6
4   U(J,K)=0.0
      U(1,1)=CT
      U(1,4)=ST*V
      U(2,2)=CT
      U(2,5)=ST*V
      U(3,3)=1.0
      U(4,1)=-ST*V
      U(4,4)=CT
      U(5,2)=-ST*V
      U(5,5)=CT

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010440
010450
010460
010470
010480
010490
010500
010510
010520
010530
010540
010550
010560
010570
010580
010590
010600
010610
010620
010630

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U(6,6)=1.0	010640
RETURN	010650
END	010660
C	010670
C	010680
SUBROUTINE STAVEC(SV,N,BC)	010690
DIMENSION BC(6,22)	010700
DO 10 J=1,6	010710
10 BC(J,N)=0.	010720
IF(SV-1.)1,1,2	010730
1 BC(4,N)=1.0	010740
BC(5,N)=1.0	010750
BC(6,N)=1.0	010760
RETURN	010770
2 IF(SV-2.)3,3,4	010780
3 BC(1,N)=1.0	010790
BC(2,N)=1.0	010800
BC(3,N)=1.0	010810
RETURN	010820
4 IF(SV-3.)5,5,6	010830
5 BC(3,N)=1.0	010840
BC(5,N)=1.0	010850
BC(6,N)=1.0	010860
RETURN	010870
6 IF(SV-4.)7,7,8	010880
7 BC(1,N)=1.0	010890
BC(3,N)=1.0	010900
BC(5,N)=1.0	010910
RETURN	010920
8 BC(2,N)=1.0	010930
BC(3,N)=1.0	010940
BC(6,N)=1.0	010950
RETURN	010960
END	010970
C	010980
C	010990
SUBROUTINE STAVEO(SV,N,BC)	011000
DIMENSION BC(6,22)	011010
DO 10 L=1,6	011020
10 BC(L,N)=0.0	011030
IF(SV-1.0)1,1,2	011040
1 BC(2,N)=1.0	011050
BC(5,N)=1.0	011060
BC(6,N)=1.0	011070
RETURN	011080
2 IF(SV-2.0)3,3,4	011090
3 BC(1,N)=1.0	011100
BC(3,N)=1.0	011110
BC(4,N)=1.0	011120
RETURN	011130
4 IF(SV-3.0)5,5,6	011140
5 BC(2,N)=1.0	011150
BC(4,N)=1.0	011160
BC(6,N)=1.0	011170
RETURN	011180
6 IF(SV-4.0)7,7,5	011190

7	BC(1,N)=1.0	011200
	BC(4,N)=1.0	011210
	BC(6,N)=1.0	011220
	RETURN	011230
	END	011240
C		011250
C		011260
	SUBROUTINE INVERT (A,N,D,L,M)	011270
C	PROGRAM FOR FINDING THE INVERSE OF A NXN MATRIX	011280
C		011290
	DIMENSION A(25,25),L(25),M(25)	011300
C	SEARCH FOR LARGEST ELEMENT	011310
	D=1.0	011320
	DO80 K=1,N	011330
	L(K)=K	011340
	M(K)=K	011350
	BIGA=A(K,K)	011360
	DO20 I=K,N	011370
	DO20 J=K,N	011380
	IF(ABSF(BIGA)-ABSF(A(I,J))) 10,20,20	011390
10	BIGA=A(I,J)	011400
	L(K)=I	011410
	M(K)=J	011420
20	CONTINUE	011430
C	INTERCHANGE ROWS	011440
	J=L(K)	011450
	IF(L(K)-K) 35,35,25	011460
25	DO30 I=1,N	011470
	HOLD=-A(K,I)	011480
	A(K,I)=A(J,I)	011490
30	A(J,I)=HOLD	011500
C	INTERCHANGE COLUMNS	011510
35	I=M(K)	011520
	IF(M(K)-K) 45,45,37	011530
37	DO40 J=1,N	011540
	HOLD=-A(J,K)	011550
	A(J,K)=A(J,I)	011560
40	A(J,I)=HOLD	011570
C	DIVIDE COLUMN BY MINUS PIVOT	011580
45	DO55 I=1,N	011590
46	IF(I-K) 50,55,50	011600
50	A(I,K)=A(I,K)/(-A(K,K))	011610
55	CONTINUE	011620
C	REDUCE MATRIX	011630
	DO65 I=1,N	011640
	DO65 J=1,N	011650
56	IF(I-K) 57,65,57	011660
57	IF(J-K) 60,65,60	011670
60	A(I,J)=A(I,K)*A(K,J)+A(I,J)	011680
65	CONTINUE	011690
C	DIVIDE ROW BY PIVOT	011700
	DO75 J=1,N	011710
68	IF(J-K) 70,75,70	011720
70	A(K,J)=A(K,J)/A(K,K)	011730
75	CONTINUE	011740
C	CONTINUED PRODUCT OF PIVOTS	011750

	D=D*A(K,K)	011760
C	REPLACE PIVOT BY RECIPROCAL	011770
	A(K,K)=1.0/A(K,K)	011780
80	CONTINUE	011790
C	FINAL ROW AND COLUMN INTERCHANGE	011800
	K=N	011810
100	K=(K-1)	011820
	IF(K) 150,150,103	011830
103	I=L(K)	011840
	IF(I-K) 120,120,105	011850
105	DO110 J=1,N	011860
	HOLD=A(J,K)	011870
	A(J,K)=-A(J,I)	011880
110	A(J,I)=HOLD	011890
120	J=M(K)	011900
	IF(J-K) 100,100,125	011910
125	DO130 I=1,N	011920
	HOLD=A(K,I)	011930
	A(K,I)=-A(J,I)	011940
130	A(J,I)=HOLD	011950
	GO TO 100	011960
150	RETURN	011970
	END	011980
C		011990
C		012000
	SUBROUTINE HANGER (CLX,CLY,CTZ,U)	012010
	DIMENSION U(6,6)	012020
	DO 1 J=1,6	012030
	DO 1 K=1,6	012040
1	U(J,K)=0.0	012050
	U(1,1)=1.0	012060
	U(2,2)=1.0	012070
	U(3,3)=1.0	012080
	U(4,4)=1.0	012090
	U(5,5)=1.0	012100
	U(6,6)=1.0	012110
	U(4,3)=CTZ	012120
	U(5,2)=-CLY	012130
	U(6,1)=CLX	012140
	RETURN	012150
	END	012160
C		012170
C		012180
	SUBROUTINE HANGE0 (CTX,CLZ,CTY,U)	012190
	DIMENSION U(6,6)	012200
	DO 1 J=1,6	012210
	DO 1 K=1,6	012220
1	U(J,K)=0.0	012230
	U(1,1)=1.0	012240
	U(2,1)=CTX	012250
	U(2,2)=1.0	012260
	U(3,3)=1.0	012270
	U(4,4)=1.0	012280
	U(5,4)=CTY	012290
	U(5,5)=1.0	012300
	U(6,3)=-CLZ	012310

U(6,6)=1.0	012320
RETURN	012330
END	012340
	012350
	012360
SUBROUTINE CFIELD (AJT,R,Z,G,SDRI,RHO,THETA,D,DI,FFAC,A)	012370
DIMENSION A(6,6)	012380
PHI=THETA/Z	012390
RHOD=RHO	012400
IF(RHO-0.0)11,11,12	012410
11 V=-1.0	012420
GO TO 1	012430
12 V=1.0	012440
1 CP=COSF(PHI)	012450
SP=SINF(PHI)	012460
W=1.0/(G *AJT)	012470
F1=(PHI*CP+SP)/2.0	012480
F3=(SP-CP*PHI)/2.0	012490
F5=(2.0-2.0*CP-PHI*SP)/2.0	012500
F6=(2.0*PHI+PHI*CP-3.0*SP)/2.0	012510
RHO=ABSF(RHO)	012520
DO 2 J=1,6	012530
DO 2 K=1,6	012540
2 A(J,K)=0.0	012550
IF(SDRI)13,13,14	012560
13 AREA=(3.141592654/4.0)*(D**2-DI**2)*FFAC	012570
SD=RHO*THETA/(G*Z*AREA)	012580
GO TO 15	012590
14 SD=0.0	012600
15 A(1,1)=CP	012610
A(1,2)=(W*RHO*F1)-(RHO*F3*R)	012620
A(1,4)=V*SP	012630
A(1,5)=V*(W+R)*(RHO*PHI*SP)/2.0	012640
A(1,6)=V*(W+R)*(RHO**2)*F3	012650
A(2,2)=CP	012660
A(2,5)=V*SP	012670
A(2,6)=V*RHO*(1.0-CP)	012680
A(3,1)=-A(2,6)	012690
A(3,2)=-1.0*A(1,6)	012700
A(3,3)=1.0	012710
A(3,4)=RHO*SP	012720
A(3,5)=((R*(RHO**2)*PHI*SP)/2.0)-W*RHO**2*F5	012730
A(3,6)=(R*RHO**3*F3)-(W*F6*RHO**3)-SD	012740
A(4,1)=-V*SP	012750
A(4,2)=-V*(W+R)*RHO*PHI*SP/2.0	012760
A(4,4)=CP	012770
A(4,5)=(R*RHO*F1)-(W*RHO*F3)	012780
A(4,6)=A(3,5)	012790
A(5,2)=-V*SP	012800
A(5,5)=CP	012810
A(5,6)=A(3,4)	012820
A(6,6)=1.0	012830
RHO=RHOD	012840
RETURN	012850
END	012860
	012870

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C
SUBROUTINE POINO (AMU,AIX,AIY,OMEGA,TL,Z,SDRI,B)
DIMENSION B(6,6)
AM=AMU*TL/(Z-1.0)
DO 1 J=1,6
DO 1 K=1,6
1 B(J,K)=0.0
B(1,1)=1.0
B(2,1)=-AM*(AIX*OMEGA)**2
B(2,2)=1.0
B(3,3)=1.0
B(4,4)=1.0
IF(SDRI-0.)9,9,10
9 B(5,4)=-AM*(AIY*OMEGA)**2
GO TO 11
10 B(5,4)=0
11 B(5,5)=1.0
B(6,3)=AM*OMEGA**2
B(6,6)=1.0
RETURN
END

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012880
012890
012900
012910
012920
012930
012940
012950
012960
012970
012980
012990
013000
013010
013020
013030
013040
013050
013060
013070
013080
013090
013100

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C
SUBROUTINE CFIELD (R,Z,G,SD,RHO,THETA,D,DI,E,A)
DIMENSION A(6,6)
PHI=THETA/Z
RD=ABSF(RHO)
IF(RHO)1,1,2
1 V=-1.0
GO TO 3
2 V=1.0
3 CP=COSE(PHI)
SP=SINF(PHI)
DO 4 J=1,6
DO 4 K=1,6
4 A(J,K)=0.0
A(1,1)=CP
A(1,2)=-V*SP
A(1,3)=-V*RD*(1.0-CP)
A(1,4)=-V*RD**2*R*(PHI-SP)
AR=3.141592654*(D**2-DI**2)/4.0
W=1.0/(E*AR)
F3=(SP-PHI*CP)*.5
F1=(PHI*CP+SP)*.5
F5=(2.0-2.0*CP-PHI*SP)*.5
F6=(2.0*PHI+PHI*CP-3.0*SP)*.5
A(1,5)=V*(RD*W*PHI*SP*.5-R*F5*RD**3)
A(1,6)=RD*W*F1+R*F6*RD**3
A(2,1)=V*SP
A(2,2)=CP
A(2,3)=RD*SP
A(2,4)=R*RD**2*(1.0-CP)
A(2,5)=RD*F3*(W+R*RD**2)
A(2,6)=A(1,5)
A(3,3)=1.0
A(3,4)=RD*R*PHI

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013110
013120
013130
013140
013150
013160
013170
013180
013190
013200
013210
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013230
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013250
013260
013270
013280
013290
013300
013310
013320
013330
013340
013350
013360
013370
013380
013390
013400
013410
013420
013430

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A(3,5)=A(2,4)	013440
A(3,6)=A(1,4)	013450
A(4,4)=1.0	013460
A(4,5)=A(2,3)	013470
A(4,6)=A(1,3)	013480
A(5,5)=CP	013490
A(5,6)=A(1,2)	013500
A(6,5)=A(2,1)	013510
A(6,6)=CP	013520
RETURN	013530
END	013540

C
C

SUBROUTINE POINT (AMU,AIY,OMEGA,TL,Z,RI,B)	
DIMENSION B(6,6)	
AM=AMU*TL/(Z-1.)	
DO 1 J=1,6	
DO 1 K=1,6	
1	B(J,K)=0.0
	B(1,1)=1.0
	B(2,2)=1.0
	B(3,3)=1.0
	IF(RI)2,2,3
2	B(4,3)=-AM*(AIY*OMEGA)**2
3	B(4,4)=1.0
	B(5,2)=AM*OMEGA**2
	B(5,5)=1.0
	B(6,1)=-B(5,2)
	B(6,6)=1.0
	RETURN
	END

C
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C

SUBROUTINE SFIELD (DI,TL,R,Z,G,SD,D,E,FFAC,A)	
DIMENSION A(6,6)	
ARA=(3.141592654/4.0)*(D**2-DI**2)	
AREA=ARA*FFAC	
DL =TL/Z	
DO 1 J=2,5	
1	A(1,J)=0.0
	A(1,1)=1.0
	A(1,6)=DL/(E*ARA)
	A(2,1)=0.0
	A(2,2)=1.0
	A(2,3)=DL
	A(2,4)=DL**2*R/2.0
	IF(SD)2,2,3
2	A(2,5)=DL**3*R/6.0-DL/(G*AREA)
	GO TO 4
3	A(2,5)=DL**3*R/6.0
4	A(2,6)=0.0
	A(3,1)=0.0
	A(3,2)=0.0


```

A(3,3)=1.0
A(3,4)=DL*R
A(3,5)=A(2,4)
A(3,6)=0.0
DO 5 J=1,6
  A(4,J)=0.0
  A(5,J)=0.0
5 A(6,J)=0.0
  A(4,4)=1.0
  A(4,5)=DL
  A(5,5)=1.0
  A(6,6)=1.0
RETURN
END

```

C
C

```

SUBROUTINE SFIELO (DI,TL,AJT,R,Z,G,SDRI,D,FFAC,A)
DIMENSION A(6,6)
AREA=(3.141592654/4.0)*(D**2-DI**2)*FFAC
DL=TL/Z
DO 2 J=1,6
  DO 2 K=1,6
2 A(J,K)=0.0
  A(1,1)=1.0
  A(2,2)=1.0
  A(3,3)=1.0
  A(4,4)=1.0
  A(5,5)=1.0
  A(6,6)=1.0
  IF(SDRI-0.)7,7,8
7 A(3,6)=DL**3*R/6.0-DL/(G*AREA)
  GO TO 1
8 A(3,6)=DL**3*R/6.0
1 A(1,2)=DL/(G*AJT)
  A(3,4)=DL
  A(3,5)=DL**2*R/2.0
  A(4,5)=DL*R
  A(4,6)=A(3,5)
  A(5,6)=DL
RETURN
END

```

C
C

```

SUBROUTINE MATMUL (A,B,U)
DIMENSION A(6,6),B(6,6),C(6,6),U(6,6)
DO 1 J=1,6
  DO 1 K=1,6
  C(J,K)=0.0
  DO 1 L=1,6
1 C(J,K)=C(J,K)+B(J,L)*A(L,K)
  DO 2 J=1,6
  DO 2 K=1,6
2 U(J,K)=C(J,K)
RETURN
END

```

C

```

014000
014010
014020
014030
014040
014050
014060
014070
014080
014090
014100
014110
014120
014130
014140
014150
014160
014170
014180
014190
014200
014210
014220
014230
014240
014250
014260
014270
014280
014290
014300
014310
014320
014330
014340
014350
014360
014370
014380
014390
014400
014410
014420
014430
014440
014450
014460
014470
014480
014490
014500
014510
014520
014530
014540
014550

```

C	SUBROUTINE FINBRA(U,VV,V,MAT,A,Z)	014560
	DIMENSION U(6,6),A(6,6),VV(6,3),V(6,3)	014570
	TYPE DOUBLE VV,V	014580
	IF(MAT) 65,64,65	014590
64	DO 67 J=1,6	014600
	DO 67 K=1,3	014610
	V(J,K)=0.	014620
	DO 67 L=1,6	014630
67	V(J,K)=V(J,K)+U(J,L)*VV(L,K)	014640
	RETURN	014650
65	N=Z-1.	014660
	DO 5 M=1,N	014670
	DO 4 J=1,6	014680
	DO 4 K=1,3	014690
	V(J,K)=0.	014700
	DO 4 L=1,6	014710
4	V(J,K)=V(J,K)+U(J,L)*VV(L,K)	014720
	DO 5 J=1,6	014730
	DO 5 K=1,3	014740
5	VV(J,K)=V(J,K)	014750
	DO 7 J=1,6	014760
	DO 7 K=1,3	014770
	V(J,K)=0.	014780
	DO 7 L=1,6	014790
7	V(J,K)=V(J,K)+A(J,L)*VV(L,K)	014800
	RETURN	014810
	END	014820
C		014830
C		014840
		014850
	SUBROUTINE FINMAT (U,UU,V,MAT,A,Z)	014860
	DIMENSION U(6,6), A(6,6),V(6,3),UU(6,3)	014870
	TYPE DOUBLE UU,V	014880
	IF(MAT) 65,64,65	014890
64	DO 67 J=1,6	014900
	DO 67 K=1,3	014910
	V(J,K)=0.	014920
	DO 67 L=1,6	014930
67	V(J,K)=V(J,K)+U(J,L)*UU(L,K)	014940
	RETURN	014950
65	N=Z-1.	014960
	DO 5 M=1,N	014970
	DO 4 J=1,6	014980
	DO 4 K=1,3	014990
	V(J,K)=0.	015000
	DO 4 L=1,6	015010
4	V(J,K)=V(J,K)+U(J,L)*UU(L,K)	015020
	DO 5 J=1,6	015030
	DO 5 K=1,3	015040
5	UU(J,K)=V(J,K)	015050
	DO 7 J=1,6	015060
	DO 7 K=1,3	015070
	V(J,K)=0.	015080
	DO 7 L=1,6	015090
7	V(J,K)=V(J,K)+A(J,L)*UU(L,K)	015100
	RETURN	015110

END	015120
	015130
	015140
SUBROUTINE BRANCH (R3,VV,PHI,U)	015150
DIMENSION R3(3,3),VV(6,3),U(6,6),R1(25,25),LL(25),MM(25),R2(3,3),	015160
IR(3,3),G1(3,3),G2(3,3)	015170
TYPE DOUBLE VV	015180
DO 4 L=1,3	015190
R1(1,L)=VV(1,L)	015200
R1(2,L)=VV(2,L)	015210
R1(3,L)=VV(3,L)	015220
R2(1,L)=VV(4,L)	015230
R2(2,L)=VV(5,L)	015240
4 R2(3,L)=VV(6,L)	015250
SP=SINF(PHI)	015260
CP=COSF(PHI)	015270
G1(1,1)=CP	015280
G1(1,2)=-SP	015290
G1(1,3)=0.	015300
G1(2,1)=SP	015310
G1(2,2)=CP	015320
G1(2,3)=0.	015330
G1(3,1)=0.	015340
G1(3,2)=0.	015350
G1(3,3)=1.	015360
G2(1,1)=1.	015370
G2(1,2)=0.	015380
G2(1,3)=0.	015390
G2(2,1)=0.	015400
G2(2,2)=CP	015410
G2(2,3)=-SP	015420
G2(3,1)=0.	015430
G2(3,2)=SP	015440
G2(3,3)=CP	015450
CALL INVERT (R1,3,D,LL,MM)	015460
DO 5 J=1,3	015470
DO 5 K=1,3	015480
R3(J,K)=0.0	015490
DO 5 L=1,3	015500
5 R3(J,K)=R3(J,K)+R1(J,L)*G1(L,K)	015510
DO 6 J=1,3	015520
DO 6 K=1,3	015530
R1(J,K)=0.0	015540
DO 6 L=1,3	015550
6 R1(J,K)=R1(J,K)+G2(J,L)*R2(L,K)	015560
DO 7 J=1,3	015570
DO 7 K=1,3	015580
R(J,K)=0.0	015590
DO 7 L=1,3	015600
7 R(J,K)=R(J,K)+R1(J,L)*R3(L,K)	015610
DO 8 J=1,6	015620
DO 8 K=1,6	015630
8 U(J,K)=0.0	015640
U(1,1)=1.0	015650
U(2,2)=1.0	015660
U(3,3)=1.0	015670

U(4,4)=1.0	015680
U(5,5)=1.0	015690
U(6,6)=1.0	015700
DO 9 L=1,3	015710
U(4,L)=R(1,L)	015720
U(5,L)=R(2,L)	015730
9 U(6,L)=R(3,L)	015740
RETURN	015750
END	015760
	015770
	015780
	015790
SUBROUTINE BRANCO (R3,VV,PHI,U)	015800
DIMENSION R3(3,3),U(6,6),VV(6,3),R1(25,25),R2(3,3),	015810
TYPE DOUBLE VV	015820
1G1(3,3),G2(3,3),S(3,3),LL(25),MM(25)	015830
DO 4 K=1,3	015840
R1(1,K)=VV(1,K)	015850
R1(2,K)=VV(3,K)	015860
R1(3,K)=VV(4,K)	015870
R2(1,K)=VV(2,K)	015880
R2(2,K)=VV(5,K)	015890
4 R2(3,K)=VV(6,K)	015900
SP=SINF(PHI)	015910
CP=COSE(PHI)	015920
G1(1,1)=CP	015930
G1(1,2)=0.	015940
G1(1,3)=-SP	015950
G1(2,1)=0.	015960
G1(2,2)=1.	015970
G1(2,3)=0.	015980
G1(3,1)=SP	015990
G1(3,2)=0.	016000
G1(3,3)=CP	016010
G2(1,1)=CP	016020
G2(1,2)=SP	016030
G2(1,3)=0.	016040
G2(2,1)=-SP	016050
G2(2,2)=CP	016060
G2(2,3)=0.	016070
G2(3,1)=0.	016080
G2(3,2)=0.	016090
G2(3,3)=1.	016100
CALL INVERT (R1,3,D,LL,MM)	016110
DO 5 J=1,3	016120
DO 5 K=1,3	016130
R3(J,K)=0.0	016140
DO 5 L=1,3	016150
5 R3(J,K)=R3(J,K)+R1(J,L)*G1(L,K)	016160
DO 6 J=1,3	016170
DO 6 K=1,3	016180
R1(J,K)=0.0	016190
DO 6 L=1,3	016200
6 R1(J,K)=R1(J,K)+G2(J,L)*R2(L,K)	016210
DO 7 J=1,3	016220
DO 7 K=1,3	016230
S(J,K)=0.0	

```

DO 7 L=1,3
7 S(J,K)=S(J,K)+R1(J,L)*R3(L,K)
DO 8 J=1,6
DO 8 K=1,6
8 U(J,K)=0.0
U(1,1)=1.0
U(2,1)=S(1,1)
U(2,2)=1.0
U(2,3)=S(1,2)
U(2,4)=S(1,3)
U(3,3)=1.0
U(4,4)=1.0
U(5,1)=S(2,1)
U(5,3)=S(2,2)
U(5,4)=S(2,3)
U(5,5)=1.0
U(6,1)=S(3,1)
U(6,3)=S(3,2)
U(6,4)=S(3,3)
U(6,6)=1.0
RETURN
END

```

C
C

```

SUBROUTINE STATEM(SVI,UU,D1,D2,P1,P2,PP,PM,SS)
DIMENSION SVI(6,1),UU(6,3)
TYPE DOUBLE UU,P1,P2,D1,D2
M=0
DO 1 J=1,6
DO 1 K=1,3
1 UU(J,K)=0.
IF(PM) 40,40,41
40 DO 42 J=1,6
IF(SVI(J,1)) 42,42,43
43 M=M+1
UU(J,M)=1.
IF(M-1) 42,44,42
44 UU(J,2)=-D1
UU(J,3)=-D2
42 CONTINUE
RETURN
41 P1=P1+D1
P2=P2+D2
IF(SS) 14,15,15
15 IF(SS-1.) 12,11,10
14 IF(PP-1.) 12,11,10
10 DO 4 J=1,6
IF(SVI(J,1)) 4,4,2
2 M=M+1
UU(J,M)=1.
IF(M-2) 4,3,5
3 UU(J,1)=P1
GO TO 4
5 UU(J,1)=P2
4 CONTINUE

```

016240
016250
016260
016270
016280
016290
016300
016310
016320
016330
016340
016350
016360
016370
016380
016390
016400
016410
016420
016430
016440
016450
016460
016470
016480
016490
016500
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016570
016580
016590
016600
016610
016620
016630
016640
016650
016660
016670
016680
016690
016700
016710
016720
016730
016740
016750
016760
016770
016780

RETURN	016790
11 DO 21 J=1,6	016800
IF(SVI(J,1)) 21,21,22	016810
22 M=M+1	016820
UU(J,M)=1.	016830
IF(M-1) 25,26,25	016840
26 UU(J,3)=P1	016850
25 IF(M-2) 21,23,21	016860
23 UU(J,3)=P2	016870
21 CONTINUE	016880
RETURN	016890
12 DO 31 J=1,6	016900
IF(SVI(J,1)) 31,31,32	016910
32 M=M+1	016920
UU(J,M)=1.	016930
IF(M-1) 35,36,35	016940
36 UU(J,2)=P2	016950
35 IF(M-2) 31,31,33	016960
33 UU(J,2)=P1	016970
31 CONTINUE	016980
RETURN	016990
END	017000
C	
C	
SUBROUTINE STATEB(SVR,VV,N,YY,L,BRC,D1,D2,PP,PM,SS,Q1,Q2,Q3,FINK)	017010
DIMENSION VV(6,3),YY(3,3,25),V(3,3),SVB(6,22)	017020
TYPE DOUBLE VV	017030
B1=0.	017040
B2=0.	017050
B3=0.	017060
D1=0.	017070
D2=0.	017080
IF(PM) 33,40,22	017090
22 IF(PRC) 21,20,21	017100
21 IF(L-1) 23,20,23	017110
23 DO 10 J=1,3	017120
DO 10 K=1,3	017130
10 V(J,K)=YY(J,K,N)	017140
IF(SS) 11,12,12	017150
11 IF(PP-1.) 32,31,30	017160
12 IF(SS-1.) 33,31,30	017170
30 B1=V(1,1)+V(1,2)*D1+V(1,3)*D2+P1	017180
B2=V(2,1)+V(2,2)*D1+V(2,3)*D2+P2	017190
B3=V(3,1)+V(3,2)*D1+V(3,3)*D2+P3	017200
GO TO 33	017210
31 B1=B1+V(1,1)*D2+V(1,2)*D1+V(1,3)*D2	017220
B2=B2+V(2,1)*D2+V(2,2)*D1+V(2,3)*D2	017230
B3=B3+V(3,1)*D2+V(3,2)*D1+V(3,3)*D2	017240
GO TO 33	017250
32 B1=B1+V(1,1)*D1+V(1,2)*D2+V(1,3)*D2	017260
B2=B2+V(2,1)*D1+V(2,2)*D2+V(2,3)*D2	017270
B3=B3+V(3,1)*D1+V(3,2)*D2+V(3,3)*D2	017280
33 Q1=Q1+B1	017290
Q2=Q2+B2	017300
Q3=Q3+B3	017310
	017320
	017330

20 M=1	017340
DO 1 J=1,6	017350
DO 1 K=1,3	017360
1 VV(J,K)=0.0	017370
DO 4 J=1,6	017380
IF(SVB(J,N)) 4,4,2	017390
2 IF(M-1) 7,3,7	017400
3 IF(PM) 34,35,35	017410
35 VV(J,1)=Q1	017420
GO TO 36	017430
34 VV(J,1)=1.	017440
36 M=M+1	017450
GO TO 4	017460
7 IF(M-2) 4,8,9	017470
8 VV(J,1)=Q2	017480
VV(J,M)=1.	017490
M=M+1	017500
GO TO 4	017510
9 VV(J,1)=Q3	017520
VV(J,M)=1.	017530
4 CONTINUE	017540
RETURN	017550
40 IF(FINK) 54,51,54	017560
54 IF(BRC) 51,51,52	017570
52 DO 53 J=1,3	017580
DO 53 K=1,3	017590
53 V(J,K)=YY(J,K,N)	017600
D1=(V(1,2)/V(1,1)+V(2,2)/V(2,1)+V(3,2)/V(3,1))/3.	017610
D2=(V(1,3)/V(1,1)+V(2,3)/V(2,1)+V(3,3)/V(3,1))/3.	017620
51 M=0	017630
DO 41 J=1,6	017640
DO 41 K=1,3	017650
41 VV(J,K)=0.	017660
DO 42 J=1,6	017670
IF(SVB(J,N)) 42,42,43	017680
43 M=M+1	017690
VV(J,M)=1.	017700
IF(M-1) 42,44,42	017710
44 VV(J,2)=-D1	017720
VV(J,3)=-D2	017730
42 CONTINUE	017740
RETURN	017750
END	017760
	017770
	017780
	017790
SUBROUTINE BRCOR(R3,UU,XX,JK,VV,PM)	017800
DIMENSION R3(3,3),UU(6,3),XX(3,3),UUU(3,3),VV(6,3)	017810
TYPE DOUBLE VV,UU	017820
IF(PM) 20,21,20	017830
20 IF(JK) 10,10,11	017840
10 DO 12 J=1,3	017850
DO 12 K=1,3	017860
12 UUU(J,K)=UU(J,K)	017870
GO TO 3	017880
11 DO 13 K=1,3	017890
UUU(1,K)=UU(1,K)	

UUU(2,K)=UU(3,K)	017900
13 UUU(3,K)=UU(4,K)	017910
3 DO 1 J=1,3	017920
DO 1 K=1,3	017930
XX(J,K)=0.	017940
DO 1 L=1,3	017950
1 XX(J,K)=XX(J,K)+R3(J,L)*UUU(L,K)	017960
RETURN	017970
21 IF(JK) 25,25,22	017980
25 DO 23 J=1,3	017990
DO 23 K=1,3	018000
23 XX(J,K)=VV(J,K)	018010
RETURN	018020
22 DO 24 K=1,3	018030
XX(1,K)=VV(1,K)	018040
XX(2,K)=VV(3,K)	018050
24 XX(3,K)=VV(4,K)	018060
RETURN	018070
END	018080
C	018090
C	018100
SUBROUTINE DELMA(UU,SVE,V,X2,Y2,KKK,X1,FF,PP,HH,RR,SS,Y1,REM,FINK,	018110
1PM,D1,D2)	018120
DIMENSION UU(6,3),V(6,3),SVE(6,1),D(3,3)	018130
TYPE DOUBLE UU,V,D,X1,X2,Y1,Y2,DV,D1,D2	018140
L=0	018150
X1=0.	018160
X2=0.	018170
Y2=0.	018180
Y1=0.	018190
D1=0.	018200
D2=0.	018210
DO 111 J=1,3	018220
DO 111 K=1,3	018230
111 D(J,K)=0.	018240
DO 91 N=1,6	018250
IF(SVE(N,1)) 91,2,91	018260
2 L=L+1	018270
DO 92 K=1,3	018280
92 V(L,K)=UU(N,K)	018290
91 CONTINUE	018300
IF(PM) 66,64,61	018310
64 IF(FINK) 63,62,63	018320
62 D1=(V(1,2)/V(1,1)+V(2,2)/V(2,1)+V(3,2)/V(3,1))/3.	018330
D2=(V(1,3)/V(1,1)+V(2,3)/V(2,1)+V(3,3)/V(3,1))/3.	018340
FINK=1.	018350
RETURN	018360
63 FINK=0.	018370
RETURN	018380
66 RETURN	018390
61 IF(SS) 311,312,312	018400
311 IF(PP-1.) 400,401,402	018410
312 IF(SS-1.) 313,314,315	018420
402 IF(RR-1.) 300,403,300	018430
315 IF(RR-1.) 340,341,340	018440
300 IF(V(3,3)) 93,94,93	018450

340 IF(V(3,2)) 93,94,93	018460
93 DO 95 J=2,3	018470
DO 95 K=1,3	018480
95 D(J,K)=V(J,K)	018490
DV=V(1,2)*D(2,3)-V(1,3)*D(2,2)	018500
IF(DV) 98,94,98	018510
94 RR=1.0	018520
HH=1.	018530
RETURN	018540
403 IF(V(2,3)) 96,200,96	018550
341 IF(V(2,2)) 96,200,96	018560
96 DO 97 K=1,3	018570
D(2,K)=V(3,K)	018580
97 D(3,K)=V(2,K)	018590
DV=V(1,2)*D(2,3)-V(1,3)*D(2,2)	018600
IF(DV) 98,200,98	018610
98 X1=X1-(V(1,1)*D(2,3)-V(1,3)*D(2,1))/DV	018620
X2=X2-(V(1,2)*D(2,1)-V(1,1)*D(2,2))/DV	018630
IF(X1) 100,102,100	018640
102 IF(X2) 100,101,100	018650
101 FF=1.0	018660
RETURN	018670
100 IF(SS) 354,355,355	018680
354 Y2=Y2-D(3,1)/D(3,3)-X1*D(3,2)/D(3,3)	018690
RETURN	018700
355 Y1=Y1-D(3,1)/D(3,2)-X2*D(3,3)/D(3,2)	018710
RETURN	018720
99 IF(SS) 20,21,21	018730
20 PP=1.	018740
GO TO 23	018750
21 SS=1.	018760
23 RR=0.	018770
HH=1.	018780
RETURN	018790
314 IF(RR=1.) 330,331,330	018800
401 IF(RR=1.0) 301,404,301	018810
330 IF(V(3,1)) 81,82,81	018820
301 IF(V(3,2)) 81,82,81	018830
81 DO 83 J=2,3	018840
DO 83 K=1,3	018850
83 D(J,K)=V(J,K)	018860
DV=V(1,1)*D(2,2)-V(1,2)*D(2,1)	018870
IF(DV) 86,82,86	018880
82 RR=1.0	018890
HH=1.	018900
RETURN	018910
331 IF(V(2,1)) 84,70,84	018920
404 IF(V(2,2)) 84,70,84	018930
84 DO 85 K=1,3	018940
D(2,K)=V(3,K)	018950
85 D(3,K)=V(2,K)	018960
DV=V(1,1)*D(2,2)-V(1,2)*D(2,1)	018970
IF(DV) 86,70,86	018980
86 X1=X1-(V(1,1)*D(2,2)-V(1,2)*D(2,3))/DV	018990
X2=X2-(V(1,1)*D(2,3)-V(1,3)*D(2,1))/DV	019000
IF(X1) 203,201,203	019010

201 IF(X2) 203,202,203	019020
202 FF=1.0	019030
RETURN	019040
203 IF(SS) 352,353,353	019050
353 Y1=Y1-D(3,3)/D(3,1)-X2*D(3,2)/D(3,1)	019060
RETURN	019070
352 Y2=Y2-D(3,3)/D(3,2)-X1*D(3,1)/D(3,2)	019080
RETURN	019090
70 IF(SS) 25,24,24	019100
24 SS=2.	019110
GO TO 26	019120
25 PP=2.	019130
26 RR=0.	019140
HH=1.	019150
RETURN	019160
313 IF(RR-1.) 320,321,320	019170
400 IF(RR-1.) 302,405,302	019180
320 IF(V(3,3)) 71,72,71	019190
302 IF(V(3,1)) 71,72,71	019200
71 DO 73 J=2,3	019210
DO 73 K=1,3	019220
73 D(J,K)=V(J,K)	019230
DV=V(1,1)*D(2,3)-V(1,3)*D(2,1)	019240
IF(DV) 76,72,76	019250
72 RR=1.0	019260
HH=1.	019270
RETURN	019280
405 IF(V(2,1)) 74,99,74	019290
321 IF(V(2,3)) 74,99,74	019300
200 KKK=1	019310
RR=0.	019320
RETURN	019330
74 DO 75 K=1,3	019340
D(2,K)=V(3,K)	019350
75 D(3,K)=V(2,K)	019360
DV=V(1,1)*D(2,3)-V(1,3)*D(2,1)	019370
IF(DV) 76,99,76	019380
76 X2=X2-(V(1,2)*D(2,3)-V(1,3)*D(2,2))/DV	019390
X1=X1-(V(1,1)*D(2,2)-D(2,1)*V(1,2))/DV	019400
IF(X1) 213,211,213	019410
211 IF(X2) 213,212,213	019420
212 FF=1.0	019430
RETURN	019440
213 IF(SS) 350,351,351	019450
351 Y1=Y1-(D(3,2)+X2*D(3,1))/D(3,3)	019460
RETURN	019470
350 Y2=Y2-(D(3,2)+X1*D(3,3))/D(3,1)	019480
RETURN	019490
END	019500

APPENDIX E

ACCURACY OF METHODS & SAMPLE PROBLEMS

E-1 General remarks.

Appendix E contains the results of vibration analyses of several piping systems. The first group was performed to establish the accuracy of the methods and compare them with the VIPIPE output. The second group was performed to establish assurance of solution by comparing the natural frequencies got from various methods.

E-2 Method accuracy analyses.

E-2-1 Straight sections.

End conditions.

System 1 - fixed-fixed.

System 2 - fixed-free.

System 3 - fixed-propped.

System parameters (for system 1,2,3).

length - 200 inches.

diameter - 2.0 inches.

wall thickness - 0.125 inches.

Intensive properties (for system 1,2,3)

density - 490 lbs/cu-ft

shear modulus - 12×10^6 psi

elastic modulus - 30×10^6 psi

Mathematical model: Distributed mass with the effect of shear deflection and rotational inertia neglected to permit comparison with classical solution.

Comparison of results.

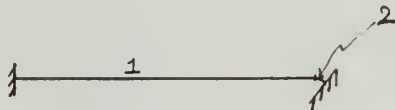
As explained in Appendix B-9-4, when the iteration reaches the solution acceptability criterion, the corresponding natural frequency is determined using linear interpolation. Theoretically exact comparison values are obtained by use of the same iteration method with the solution acceptability criterion as 0.0007 rad/sec. The values obtained using VIPIPE have a solution acceptability criterion 0.0185 rad/sec. Comparison is made by percent difference.

The M-D method fails the systems 1 through 3, because the purging factors turn out to be all zeros. So for the purpose of testing the M-D method, modified System 1 is provided below.

The letters D or S in parentheses denote double precision and single precision respectively.

Modified System 1:

Add a very small curved section to the System 1. Second section is a mathematical model which has the same property as System 1 except the system parameters.



Second section parameters:

included angle of arc - 45 degrees

radius of curvature - 0.01 inches

(This value of radius of curvature is not possible in an actual system, since the radius of curvature less than the radius of the pipe. In effect subroutine STIFCO or STIFOO will provide the transfer matrix of a massless rigid corner).

Results and comparison.*

a. System 1 and modified System 1.** (in-plane).

Mode No.	Freq.(rad/sec) and difference in %.			
	Comparison values	Delta m. (D)	P-M.-A(D)	P-Mm.-F(D)
1	75.09994540	75.09994544 0 %	75.09994550 0 %	75.09994571 0 %
2	207.01589137	207.01589036 $5.0 \times 10^{-7} \%$	207.01589094 0 %	207.01589228 $4.4 \times 10^{-7} \%$
3	405.83391916	405.83393183 $3.0 \times 10^{-6} \%$	405.83403022 $2.4 \times 10^{-5} \%$	405.83394702 $7.0 \times 10^{-7} \%$
4	670.86408383	670.86310856 $2.8 \times 10^{-4} \%$	670.86341454 $1.0 \times 10^{-5} \%$	670.86345124 $9.1 \times 10^{-5} \%$
5	1002.15520225	1002.19978814 $4.7 \times 10^{-3} \%$	1002.18735784 $3.2 \times 10^{-3} \%$	1002.19300449 $3.8 \times 10^{-3} \%$
6	1399.70436269		1399.59981662 $7.5 \times 10^{-3} \%$	1399.64448571 $4.3 \times 10^{-3} \%$
7	1863.51172599			1959.59556134 5.1 %

Mode No.	Freq.(rad.sec) and difference in %			
	Delta m. (S)	P-M m.-A(S)	M-D m.(D)***	M-D m.(S)***
1.	75.09994535 0 %	75.09994865 $4.0 \times 10^{-6} \%$	75.09994539 0 %	75.09994534 0 %
2	207.01589228 $5.0 \times 10^{-8} \%$	207.01589071 0 %	207.01588836 $1.5 \times 10^{-6} \%$	207.01588891 0 %
3	405.83398145 $1.5 \times 10^{-6} \%$	405.83403455 $2.1 \times 10^{-5} \%$	405.83395151 $9.5 \times 10^{-6} \%$	405.83401120 $2.0 \times 10^{-5} \%$
4	670.86345124 $7.5 \times 10^{-5} \%$	670.86256288 $2.2 \times 10^{-4} \%$	670.86374475 $5.1 \times 10^{-5} \%$	670.86118054 $4.3 \times 10^{-4} \%$
5		1002.23499946 $8.0 \times 10^{-3} \%$	1002.19909306 $4.4 \times 10^{-4} \%$	1002.21779856 $6.3 \times 10^{-3} \%$
6		1399.17084415 $4.5 \times 10^{-2} \%$	1399.94044241 $3.4 \times 10^{-2} \%$	

*Sources of comparison frequencies for system 1 through 3 are given in E-2-2.

(Additional footnotes on next page).

****In all the tables which follow, simple and evident abbreviations are used.**
For example, P-M m.-A(D) means the P-M method, sub-procedure A in double precision arithmetic.

*****Modified System 1.**

b. System 2.(in-plane)

Mode NO	Freq.(rad/sec) and difference in %			
	Comparison values	Delta m. (D)	P-M m.-A (D)	P-M m.-F (D)
1	11.80213586	11.80213389 $5.5 \times 10^{-5} \%$	11.80214614 $9.5 \times 10^{-5} \%$	11.80213746 $1.4 \times 10^{-5} \%$
2	73.96272297	73.96272167 $1.7 \times 10^{-6} \%$	73.96272478 $1.7 \times 10^{-6} \%$	73.96272367 0 %
3	207.09776595	207.09776685 $4.1 \times 10^{-8} \%$	207.09776388 $1.1 \times 10^{-6} \%$	207.09776562 0 %
4	405.82896584	405.82894132 $6.1 \times 10^{-6} \%$	405.82893748 $7.0 \times 10^{-6} \%$	405.82900191 $9.1 \times 10^{-6} \%$
5	670.86435913	670.86200817 $2.2 \times 10^{-5} \%$	670.86326697 $1.7 \times 10^{-5} \%$	670.86325735 $1.7 \times 10^{-5} \%$
6	1002.1551877	1002.26833752 $1.1 \times 10^{-2} \%$	1002.18809484 $4.4 \times 10^{-3} \%$	1002.19715557 $4.2 \times 10^{-3} \%$
7	1399.70436329	1399.08849534 $2.0 \times 10^{-2} \%$	1399.61231065 $6.6 \times 10^{-3} \%$	1399.14599562 $4.2 \times 10^{-3} \%$
8	1863.51172626		1862.55983728 $5.1 \times 10^{-2} \%$	

c. System 3.(in-plane)

Mode NO	Freq.(rad/sec) and difference in %			
	Comparison values	Delta m. (D)	P-M m.-A (D)	P-M m.-F (D)
1	51.75397285	51.75397263 0 %	51.75397569 0 %	51.75397564 0 %
2	167.71502090	167.71602009 0 %	167.71602200 0 %	167.71602189 0 %
3	349.92609071	349.92608231 $1.1 \times 10^{-7} \%$	349.92610259 $3.4 \times 10^{-6} \%$	349.92610209 $3.4 \times 10^{-6} \%$
4	598.39432089	598.39381063 $8.5 \times 10^{-5} \%$	598.39446341 $2.5 \times 10^{-5} \%$	598.39407344 $4.2 \times 10^{-5} \%$
5	913.12074582	913.13150640 $1.2 \times 10^{-3} \%$	913.13180993 $1.2 \times 10^{-3} \%$	913.13109335 $1.2 \times 10^{-3} \%$
6	1294.10536551	1293.77838653 $2.7 \times 10^{-2} \%$	1294.11223844 $5.8 \times 10^{-4} \%$	1294.11761177 $1.9 \times 10^{-4} \%$
7	1741.34817946		1748.59011823 $4.2 \times 10^{-1} \%$	

E-2-2 Sources of comparison frequencies for system 1 through 3.

The governing fourth order equation of a homogeneous single component straight system

$$\frac{d^4 y}{dx^4} = \frac{\mu \omega^4}{E \cdot I} \cdot y$$

was solved to give

$$y = a \cos \alpha x + b \cosh \alpha x + c \sin \alpha x + d \sinh \alpha x$$

Upon applying boundary conditions for System 1 ($y = y' = 0$ at $x = 0$, $x = L$) and simplifying, it was found that the eigenvalues of the system must satisfy

$$\cos \alpha \cdot \cosh \alpha = 1 \quad (\text{E-2-2-1})$$

where

$$\alpha = \sqrt[4]{\frac{\mu \omega^4}{E I}} \cdot L \quad (\text{E-2-2-2})$$

Upon applying the boundary conditions for System 2 ($y = y' = 0$ at $x = 0$, $y'' = y''' = 0$ at $x = L$), it was found that the eigenvalues of the system must satisfy

$$\cos \alpha \cdot \cosh \alpha = -1 \quad (\text{E-2-2-3})$$

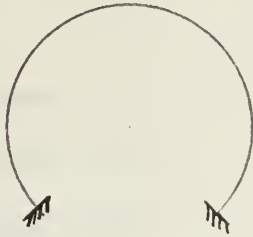
Upon applying the boundary conditions of System 3 ($y = y' = 0$ at $x = 0$, $y = y'' = 0$ at $x = L$), it was found that the eigenvalues must satisfy

$$\sin \alpha \cdot \cosh \alpha - \cos \alpha \cdot \cosh \alpha = 0 \quad (\text{E-2-2-4})$$

The roots of equation E-2-2-1, E-2-2-3 and E-2-2-4, from which the natural frequencies of the corresponding systems may easily be found, were obtained through iteration using a digital computer.

E-2-3 Curved section (System 4)

System parameters:



radius of curvature 50 inches
included angle of arc 270 degrees
diameter 2.0 inches
wall thickness .125 inches

Intensive properties:

density 556. lbs/cu-ft
shear modulus 12×10^6 psi
elastic modulus 30×10^6 psi

End conditions: fixed-fixed.

Mathematical model: lumped mass with shear deflection and rotational inertia neglected.

Results.

a. In-plane vibration*

Mode No.	Frequency (rad/sec)			
	Delta m. (D)	M-D m. (D)	P-M m.-A (D)	P-M m.-C (D)
1	66.69934902	66.69934902	66.69934980	66.69934948
2	171.22352538	171.22352538	171.22352610	171.22352592
3	335.35007083	335.35007083	335.35007083	335.35007174
4		535.64529414	535.6429665	535.64529637
5		773.44358487	773.44358842	773.44358812

b. Out-of-plane vibration*

Mode No.	Frequency (rad/sec)		
	Delta m. (D)	M-D m. (D)	P-M m.-A (D)
1	35.36309735	35.36309735	35.36309778
2	94.42588921	94.42588921	94.42588653
3	207.89287257	207.89278647	207.89287345
4	369.11199342	369.11278647	369.11278689
5		571.22235666	571.22235577

*See footnote next page.

*The fundamental frequency value of in-plane and out-of-plane case are close to the values obtained by employing equation $\omega = F(\alpha) \cdot (EI/\mu R^4)^{\frac{1}{2}}$ after obtaining $F(\alpha)$ from the curves in reference [5]. Natural frequency values from the three methods are the same up to six digits. One can get more higher mode frequencies using the M-D and P-M method.

E-3 Other Sample Problems

A. System 5

System parameters

Parameter	component						
	1	2	3	4	5	6	7
diameter (in)	2.0	2.0	2.0	H A	2.0	2.0	2.0
wall thickness (in)	0.75	0.75	0.75	N G	0.75	0.75	0.75
length (in)	130.	1.40	6.0	E R	60.21	1.50	6.28
radius of curve (in)		4.0			30.		4.0
incl. angle of arc		20.			115.		90.

Intensive properties of all components except hangers:

density: 556 lbs/cu-ft.*

shear modulus: 12×10^6 psi

elastic modulus: 30×10^6 psi

Hanger spring rates:

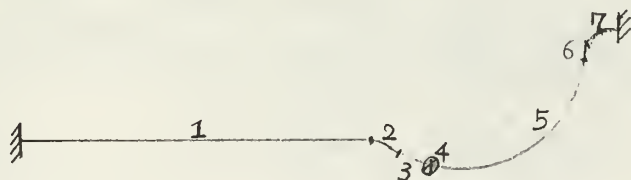
CLX, CLY, CLZ 1000lbs/in.

CTX, CTY, CTZ 1000 in-lb/rad.

End conditions:

both ends of system fixed.

Mathematical model: Distributed mass with the effects of shear deflection and rotational inertia considered.



*This fictitious value was chosen to permit comparing with results of Fink [4] who used the same value.

Results. (in-plane vibration)

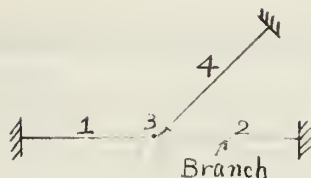
Mode NO	Frequency(rad/sec)			
	Delta m. (D)	M-D m. (D)	P-M m.-A (D)	P-M m.-C (D)
1	116.89479180			
2	304.88354398			
3	566.43670976			
4		826.88282292	826.88281950	826.88281929
5		1126.83773518	1126.83799669	1126.83797508
6		1666.58935151	1666.58812881	1666.58878192
7		2099.54173011	2099.5352509	2099.52844030

Results in single precision arithmetic and comparison with double precision results.

Mode NO	Frequency(rad/sec) and diff. in %			
	Delta m. (S)	M-D m. (B)	P-M m.-A (S)	P-M m.-C (S)
1	116.89479181 0 %			
2	304.88354658 1.0×10^{-6} %			
3	566.43620786 8.9×10^{-5} %			
4		826.88281494 1.0×10^{-6} %	826.88281417 6.0×10^{-7} %	826.88281764 1.9×10^{-7} %
5		1126.83768564 4.4×10^{-6} %	1126.83786333 1.2×10^{-5} %	1126.83745223 4.6×10^{-5} %
6		1666.58927426 4.7×10^{-6} %		
7		2099.53091794 5.0×10^{-3} %		

B. System 6 (single branch system)

System parameters:



parameter	components			
	1	2	3	4
dia. (inch)	2.0	2.0	2.0	2.0
wall thickness (inch)	.125	.125	.125	.125
length (inch)	100.	100.	.01	100.
incl. angle of arc. (degree)	0	0	45	0

Intensive properties:

density 556 lbs/cu-ft

shear modulus 12×10^6 psi

elastic modulus 30×10^6 psi

End conditions: All ends fixed.

Mathematical model: Distributed mass with the effects of shear deflection and rotational inertia neglected.

Results

Mode No.	Frequency (rad/sec)			
	Delta m. (D)	M-D m. (D)	P-M m. -A (D)	P-M m. -C (D)
1	194.25470558			
2	278.61078241			
3	628.59556355			
4	747.68138190	747.68087208	747.68137310	747.68137759
5		1300.29478437	1300.31646085	1300.31641281
6		1381.79339427	1381.86234251	1381.86230734
7		1991.80356348	1991.38946110	1991.38946101
8		2257.92926800	2257.58161443	2257.58016324

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		2b. GROUP N.A.	
3. REPORT TITLE Vibration Analysis of Piping Systems			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) None			
5. AUTHOR(S) (Last name, first name, initial) KIM, Yong Su, LT, ROKN			
6. REPORT DATE May 1966	7a. TOTAL NO. OF PAGES 130	7b. NO. OF REFS 7	
8a. CONTRACT OR GRANT NO. N.A.	9a. ORIGINATOR'S REPORT NUMBER(S) N.A.		
b. PROJECT NO. N.A.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) N.A.		
c. N.A.			
d.			
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13. ABSTRACT A digital computer program, originally developed by George E. Fink, capable of determining the in-and/or out-of-plane vibration frequencies of a single plane piping system, using the basic transfer method, is modified and augmented by the writer. The program is modified to use the writer's method to reduce the amount of calculation and is augmented to use Marguerre and Uhrig's modifying frequency determinant method and Pestel and Mahrenholtz's remainder method for higher natural frequencies. An analysis is made of difficulties encountered with the original transfer method and utilizing the modified methods. The program accepts branched systems and non-still intermediate supports. A discussion and an explanation of how to use the program are included.			

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Piping Systems						
Vibration Analysis						
Computer Program						
Transfer Matrix						
Natural Frequencies						

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